Fractional Calculus, Delay Dynamics and Networked Control Systems

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Outline

- CSOIS (Center for Self-Organizing & Intelligent Systems)
- Fractional Calculus and Fractional Order Thinking
- Delay Dynamics
- Networked Control Systems
- Concluding Remarks

Utah State University

Located in Logan, Utah, USA
80 miles North of Salt Lake City

22,000 students study at USU's Logan campus, nestled in the Rocky Mountains of the inter-mountain west

CSOIS is a research center in the Department of Electrical and Computer Engineering

CENTER FOR SELF-ORGANIZING AND INTELLIGENT SYSTEMS

CSOIS Core Capabilities and Expertise

- Control System Engineering
  - Algorithms (Intelligent Control)
  - Actuators and Sensors
  - Hardware and Software Implementation
- Intelligent Planning and Optimization
- Real-Time Programming
- Electronics Design and Implementation
- Mechanical Engineering Design and Implementation
- System Integration

We make real systems that WORK!

Selected CSOIS Research Strengths

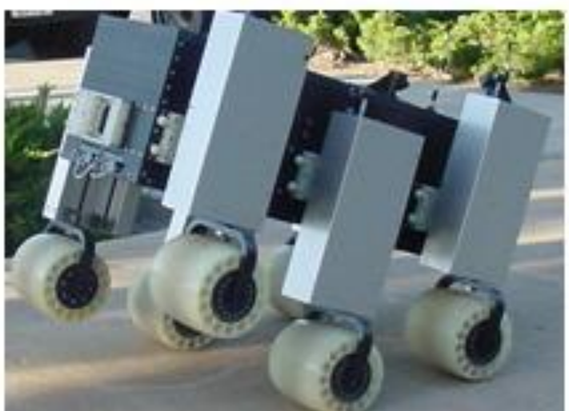
- ODV (omni-directional vehicle) Autonomous Robotics
- Iterative Learning Control Techniques
- MAS-net (mobile actuator and sensor networks)
- Smart Mechatronics, Computer Vision, Multi-UAV-Based Collaborative Remote Sensing, Multispectral Imager
- Unmanned Autonomous Vehicles (UAVs); Cooperative Control; Formation Control; Information Consensus; Engineered Swarms
- <u>Fractional Dynamical Systems, Fractional Order Signal Processing and Fractional Order Control</u>

Some Robots Built At USU

T1

T2

ODIS



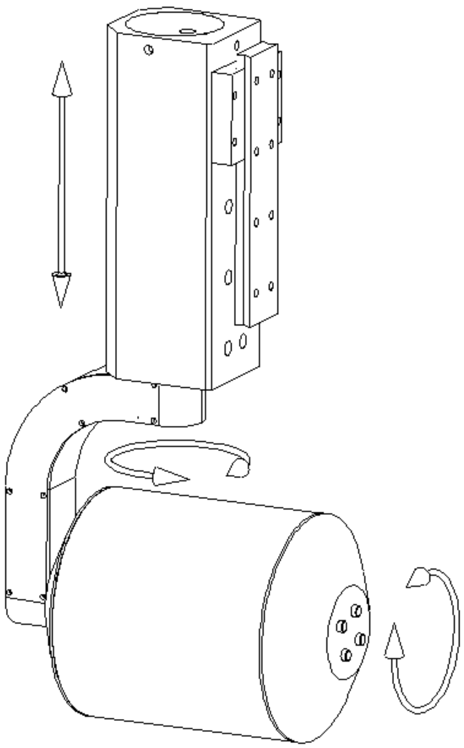
T3

Autonomous Vehicle Technology

- Autonomous vehicles are enabled by advances in:
  - Vehicle concept and mechanical design
  - Vehicle electronics (vetronics)
  - Sensors (e.g., GPS)
  - Control
  - Planning
- We consider two key aspects of autonomy:
  - Inherent mobility capability built into the vehicle
  - Mobility control capability to exploit these capabilities

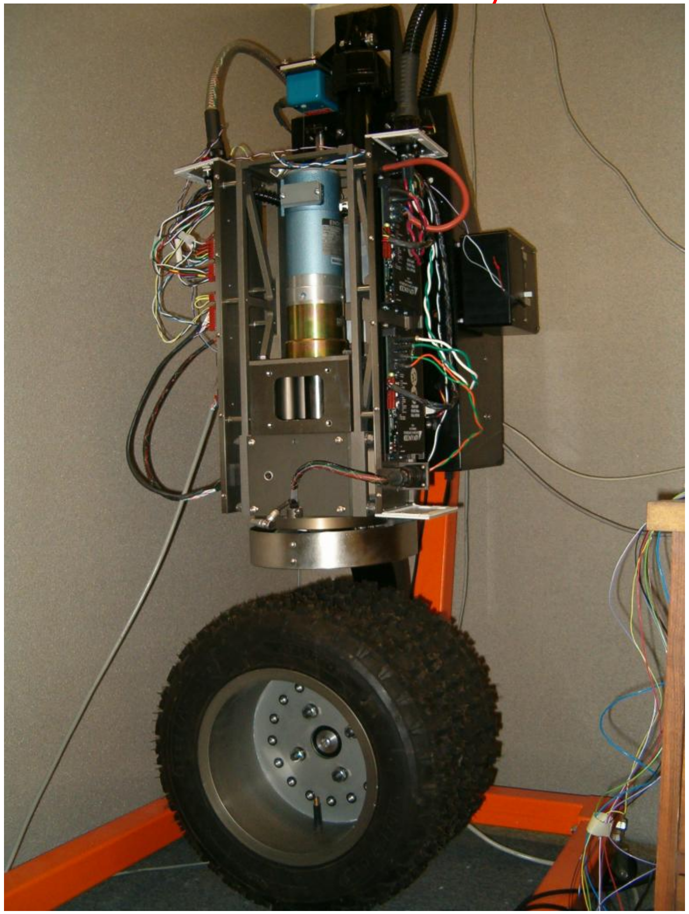
USU ODV Technology

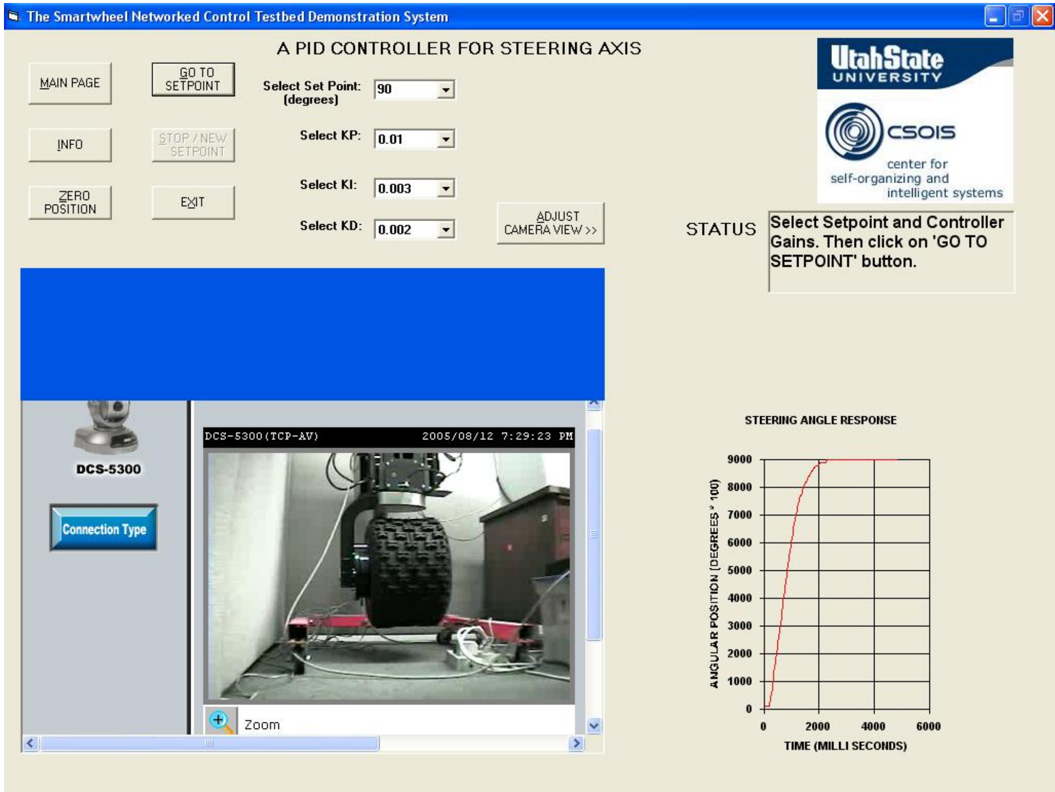
- USU has worked on a mobility capability called the “smart wheel”
- Each “smart wheel” has two or three independent degrees of freedom:
  - Drive
  - Steering (infinite rotation)
  - Height
- Multiple smart wheels on a chassis creates a “nearly-holonomic” or omni-directional (ODV) vehicle



USU Smart Wheel Demo Rig

"Omni-directional Robotic Wheel - A Mobile Real-Time Control Systems Laboratory", IJEE 2008.





http://www.csois.usu.edu/people/smartwheel/CompleteInfoPage.htm

USU-Developed Robot Family

ODIS On Duty in Baghdad

"Putting Robots in Harm's Way, So People Aren't"

CSOIS Robotics/Control Research

- Initial focus on automation and control
- Later, significant program aimed at single-entity autonomous mobile robots
  - Hardware development
  - Software architectures/algorithms for autonomy
  - Led to commercialized robot

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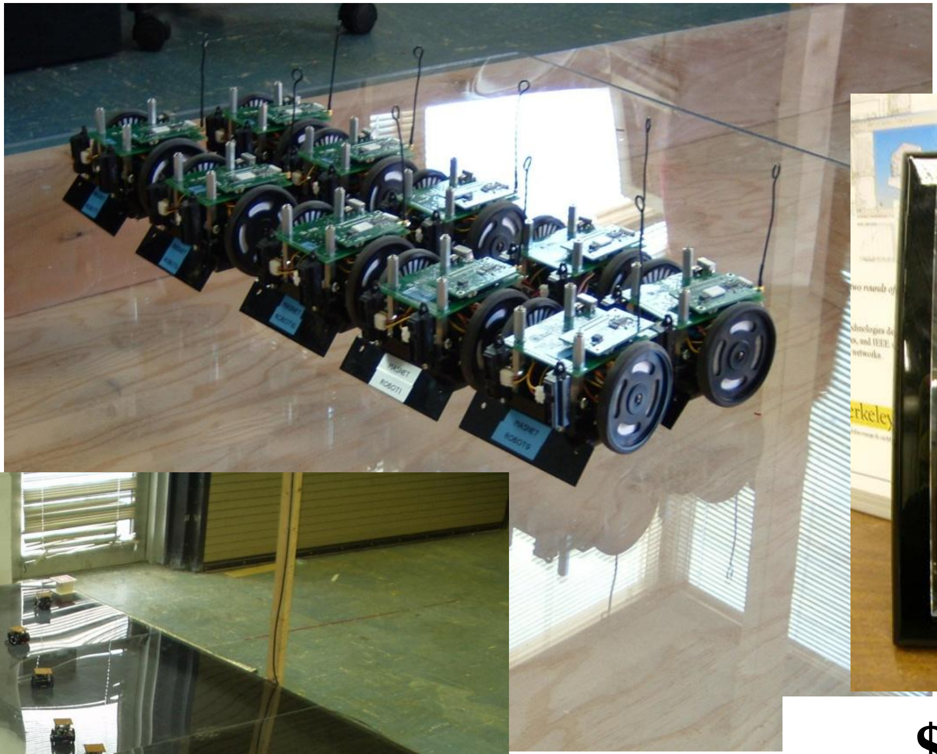
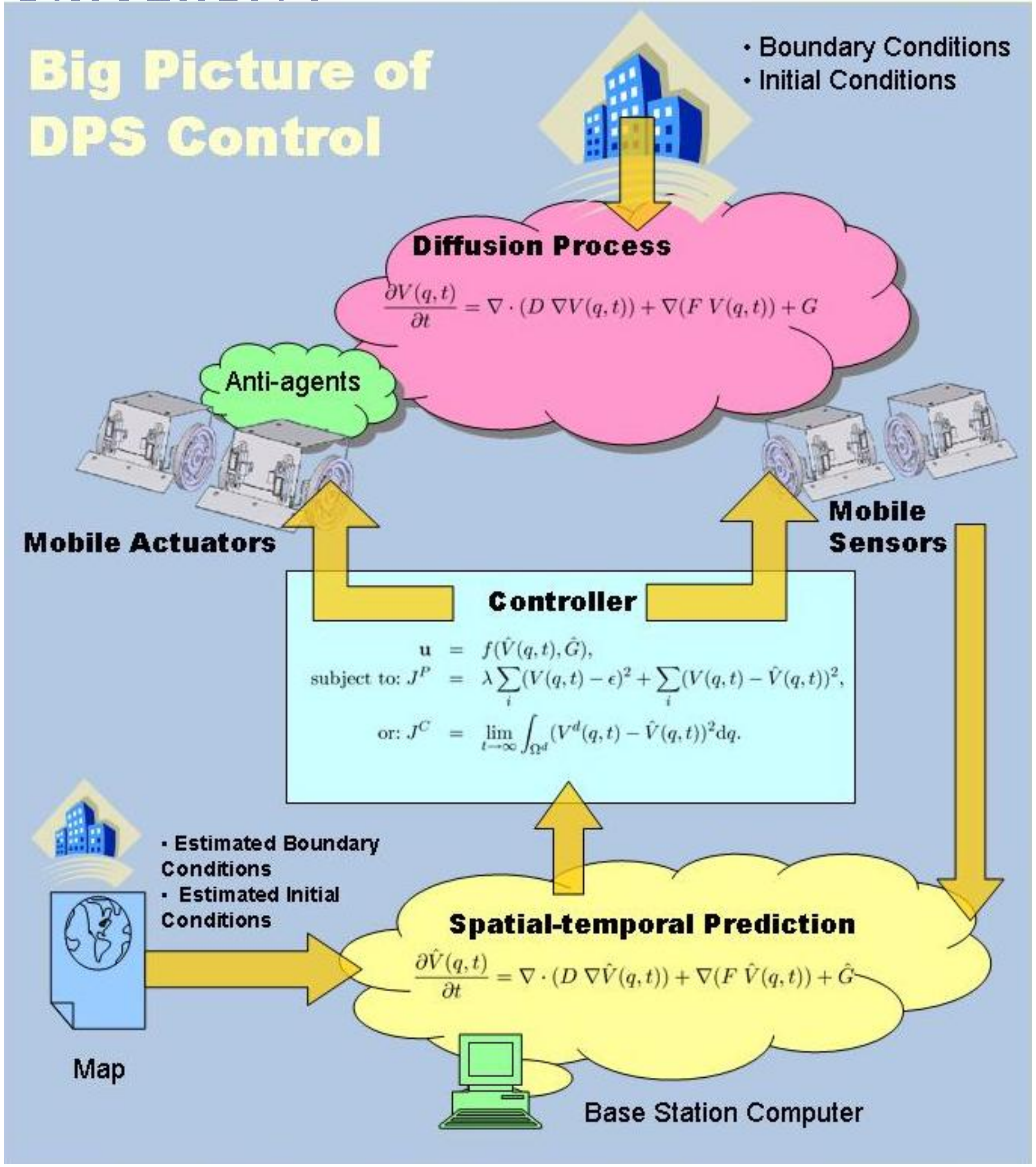
- Current focus on network of multiple, cooperating mobile robots
  - Leads to MAS-net idea (Mobile Actuator/Sensor Networks)

Mote-Based Distributed Robots

Prototype plume-tracking testbed - 2004



$2000 2nd Place Prize in 2005 Crossbow Smart-Dust Challenge



DPS:
distributed parameter systems

Features:

- Domain of interest
- Sensor configuration
- Sensor effective region
- Actuator configuration
- Actuator effective region
- Mobile or static
- Communicating or not
- Collocated or not

MAS-net Project:

Smart Sniffing and Spraying Problem

Sensors and actuators are all mobile

CSOIS + UWRL = UAV + Water

Multi UAV-Based Collaborative Multispectral Remote Sensing System for Optimal Real-Time Irrigation Control, Water Resource Management, and Ecological Inferential Measurement

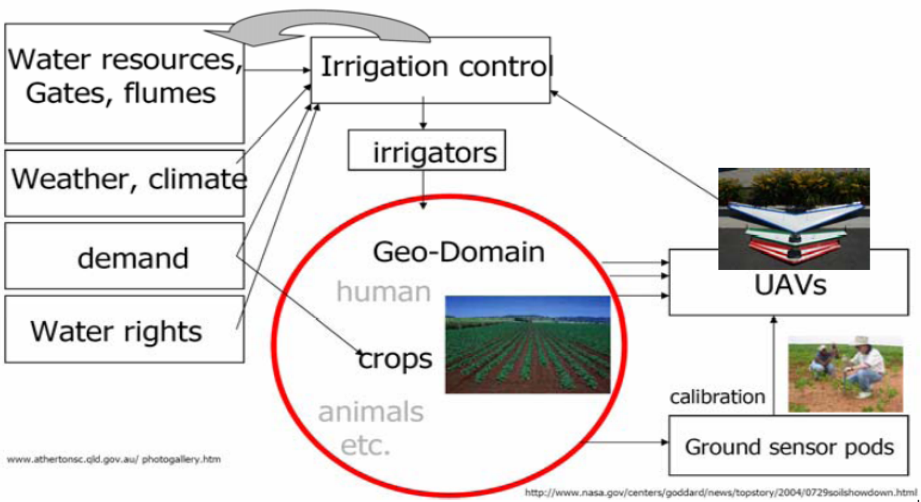
Why UAV (Unmanned Aerial Vehicle)?

- Existing remote sensing – spatial resolutions too coarse not suitable for irrigation control. The desirable scale for some applications is ~1-meter scale;
- In situ mesh networked sensors. OK. But not sustainable (battery issue, communication issues, obstruction to farming machines …)
- Placing networked sensor pods. OK. But labor intensive. Resolution/coverage is not enough
- Not flexible or active to tell which part of the land is "*thirsty*" or under water stress.

Why Water?

- Irrigated agriculture uses a large fraction of the fresh water resources around the world. Irrigated agriculture uses more water than all other sectors together.
- Small saving in agriculture will save relatively large amount of water for other sectors.
- Huge societal and economical impacts, not only for Utah but also for the dry world.
- Peripheral economic growth in Utah due to improved water access. In Utah, diversions for irrigated agriculture represent approximately 85 percent of the state's water use.
- To achieve greater efficiencies in the agricultural sector, better and timelier information is needed by canal and reservoir operators, farmers, and other water managers.
- The cost of acquiring and disseminating this information must be kept low. This is a common problem in arid regions around the world.

WaterWatch?



Contact: Dr. YangQuan Chen, **T:** (435)797-0148;
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UAV Platforms

Platform OSAM-UAV (open source autonomous multi)

Hardware

- CPD4SENUNIT: IR Sensor (2cm×2cm×0.6cm, 10g): 2-axis infrared sensor
- Tiny13 v1.1: Processor board (6.5cm×3.5cm, 25g): ARM7 CPU, u-blox LEA-4P GPS receiver, Chipset, etc.

Software

- Ground Station:
  - Data Display
  - Cockpit GUI
  - Control Gain Tuning
  - Waypoint Control
  - Multi-Vehicle Coordination
  - Completely Open Source

Main features of OSAM-UAV

Paparazzi is a complete system of hardware and software for autonomous aircraft as well as complete ground station mission planning and monitoring software utilizing a bi-directional datalink for telemetry and control.

Wingspan 48"-100"; Payload: 0.5-2.5lb; Communication range: 30 miles;
Battery endurance: 30+ minutes, dependent on airframe.
Completely Open Source; ARM7: 60 Mhz, 40KB RAM, 512KB Flash

Fig. 1 Tiny13 v1.1

Fig. 2 IR Sensor

Fig. 3 Ground Station

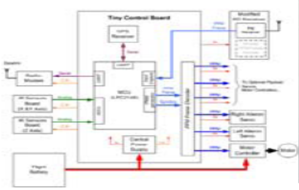
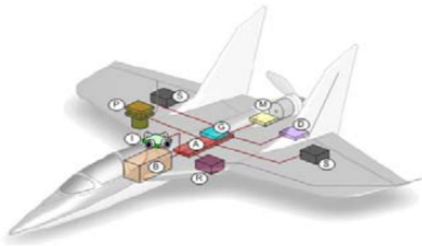


Fig. 4 Architecture

Fig. 5 Airframes (100", 60", 48")

Fig. 6 The whole system



- Autopilot Control Board
- Battery
- Datalink Radio-Modem & Antenna
- GPS Receiver
- IR Sensor Board
- Motor & Controller
- RC Receiver & Antenna
- Servos
- Payload = Camera & Video Transmitter

Paparazzi OS Project: http://www.recherche.enac.fr/paparazzi/wiki/index.php/Main_Page

OSAM UAV Team won 2nd @ AUVSI UAS Competition, June 2008

http://www.engr.usu.edu/wiki/index.php/OSAM

Utah State – Wins $8,000 for 2nd Place Overall, 2nd Place in Mission, Honorable Mention in both Orals and Journal, and Prize Barrels for Autonomous Mission Flight, Autonomous Landing, JAUS and Perfect Identification of the Off-Path Target.

UtahSt UNIVERS

CSOIS CENTER FOR SELF-ORGANIZING AND INTELLIGENT SYSTEMS

- We won #1 in AUVSI 2009 UAS Competition!!
  – June 17-21, 2009. Maryland AFB.
  – $14000 cash award.
  – Other registered participants: UCSD, MIT, Cornell, NCSU etc.
  – It will make some headlines!
  – We are the second time to participate this event!
  – UCSD, Embry Riddle,Cornell, U Alberta, UT Austin.

Current Foci

- Multi-UAV-based Band-Reconfigurable Multi-spectral Collaborative "Personal Remote Sensing"
- Fractional Order Control for Industrial Applications (hard disk drives et al.)
- MAS-net as/for CPS (Cyber-Physical System)
- Fractional Signal Processing Techniques for Applications related to Bio-X

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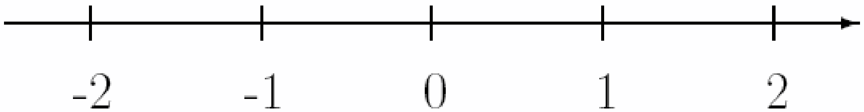
… from integer to non-integer …



$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n}$$

$$x^n = e^{n \ln x}$$

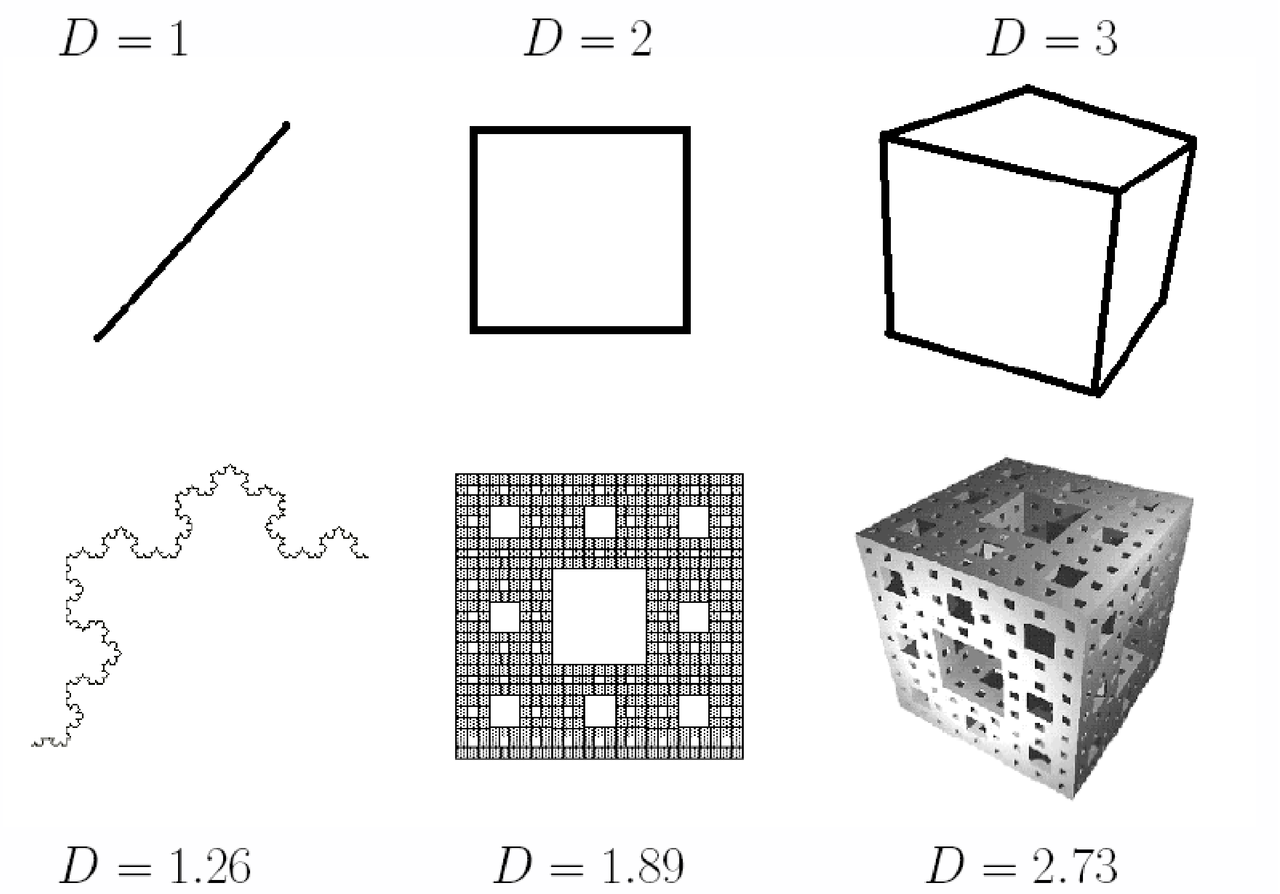
$$n! = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot (n-1) \cdot n,$$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \qquad x > 0,$$

$$\Gamma(n+1) = 1 \cdot 2 \cdot 3 \cdot \ \ldots \cdot n = n!$$

Slide credit: Igor Podlubny

… from integer to non-integer …



Slide credit: Igor Podlubny

Interpolation of operations

$$f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \frac{d^3 f}{dt^3}, \quad \ldots$$

$$f, \quad \int f(t)dt, \quad \int dt \int f(t)dt, \quad \int dt \int dt \int f(t)dt, \quad \ldots$$

$$\ldots, \quad \frac{d^{-2} f}{dt^{-2}}, \quad \frac{d^{-1} f}{dt^{-1}}, \quad f, \quad \frac{df}{dt}, \quad \frac{d^2 f}{dt^2}, \quad \ldots$$

Slide credit: Igor Podlubny

"Fractional Order Thinking" or, "In Between Thinking"

- For example
  - Between integers there are non-integers;
  - Between logic 0 and logic 1, there is the fuzzy logic;
  - Between integer order splines, there are "fractional order splines"
  - Between integer high order moments, there are noninteger order moments (e.g. FLOS)
  - Between "integer dimensions", there are fractal dimensions
  - Fractional Fourier transform (FrFT) – in-between time-n-freq.
  - Non-Integer order calculus (fractional order calculus – abuse of terminology.) (FOC)

Fractional Calculus was born in 1695

$$\frac{d^n f}{dt^n}$$

What if the order will be $n = ½$?

It will lead to a paradox, from which one day useful consequences will be drawn.

G.F.A. de L'Hôpital
(1661–1704)

G.W. Leibniz
(1646–1716)

Slide credit: Igor Podlubny

G. W. Leibniz (1695–1697)

In the letters to J. Wallis and J. Bernulli (in 1697) Leibniz mentioned the possible approach to fractional-order differentiation in that sense, that for non-integer values of n the definition could be the following:

$$\frac{d^n e^{mx}}{dx^n} = m^n e^{mx},$$

L. Euler (1730)

$$\frac{d^n x^m}{dx^n} = m(m-1)\ldots(m-n+1)x^{m-n}$$

$$\Gamma(m+1) = m(m-1)\ldots(m-n+1)\,\Gamma(m-n+1)$$

$$\frac{d^n x^m}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n}.$$

Euler suggested to use this relationship also for negative or non-integer (rational) values of n. Taking $m=1$ and $n=\frac{1}{2}$, Euler obtained:

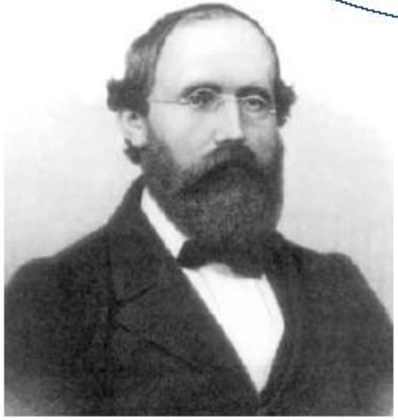
$$\frac{d^{1/2}x}{dx^{1/2}} = \sqrt{\frac{4x}{\pi}} \qquad \left(= \frac{2}{\sqrt{\pi}}x^{1/2}\right)$$

Slide credit: Igor Podlubny

Riemann–Liouville definition

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt} \right)^n \int_a^t \frac{f(\tau)\, d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ (n-1 \le \alpha < n) $$



G.F.B. Riemann
(1826–1866)

J. Liouville
(1809–1882)

Slide credit: Igor Podlubny

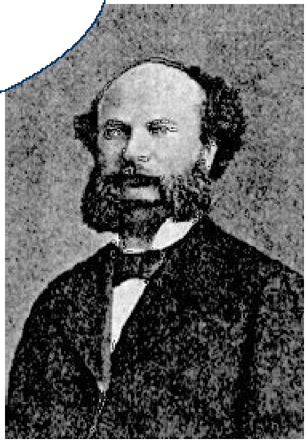
Grünwald–Letnikov definition

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh) $$

$[x]$ – integer part of x

A.K. Grünwald

A.V. Letnikov



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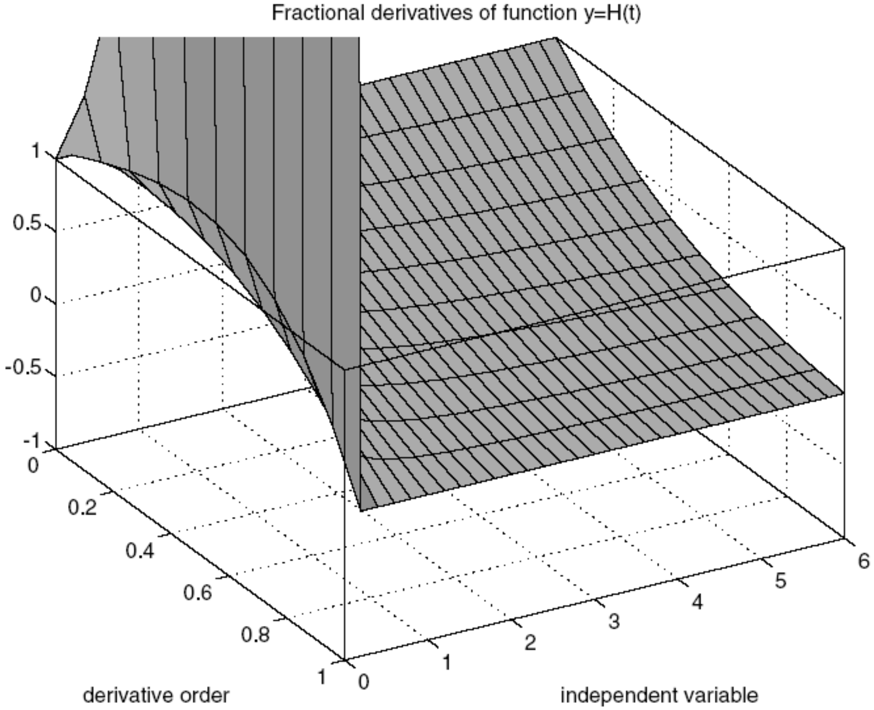
Operator ${}_a\mathrm{D}_t^\alpha$

A generalization of differential and integral operators:

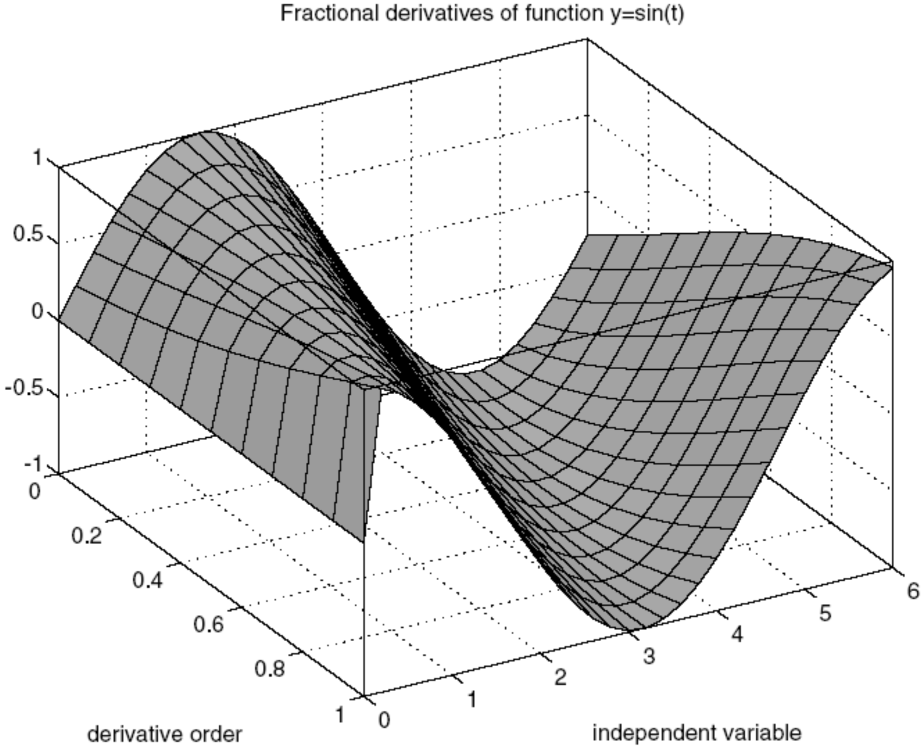
$$
{}_a\mathrm{D}_t^\alpha = \begin{cases} \mathrm{d}^\alpha/\mathrm{d}t^\alpha & \mathbb{R}(\alpha) > 0, \\ 1 & \mathbb{R}(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha} & \mathbb{R}(\alpha) < 0. \end{cases} \tag{7}
$$

There are two commonly used definitions for the general fractional order differentiation and integral, i.e., the **Grünwald-Letnikov definition** and the **Riemann-Liouville definition**.

Example: Heaviside's unit step



Example: $\sin(t)$



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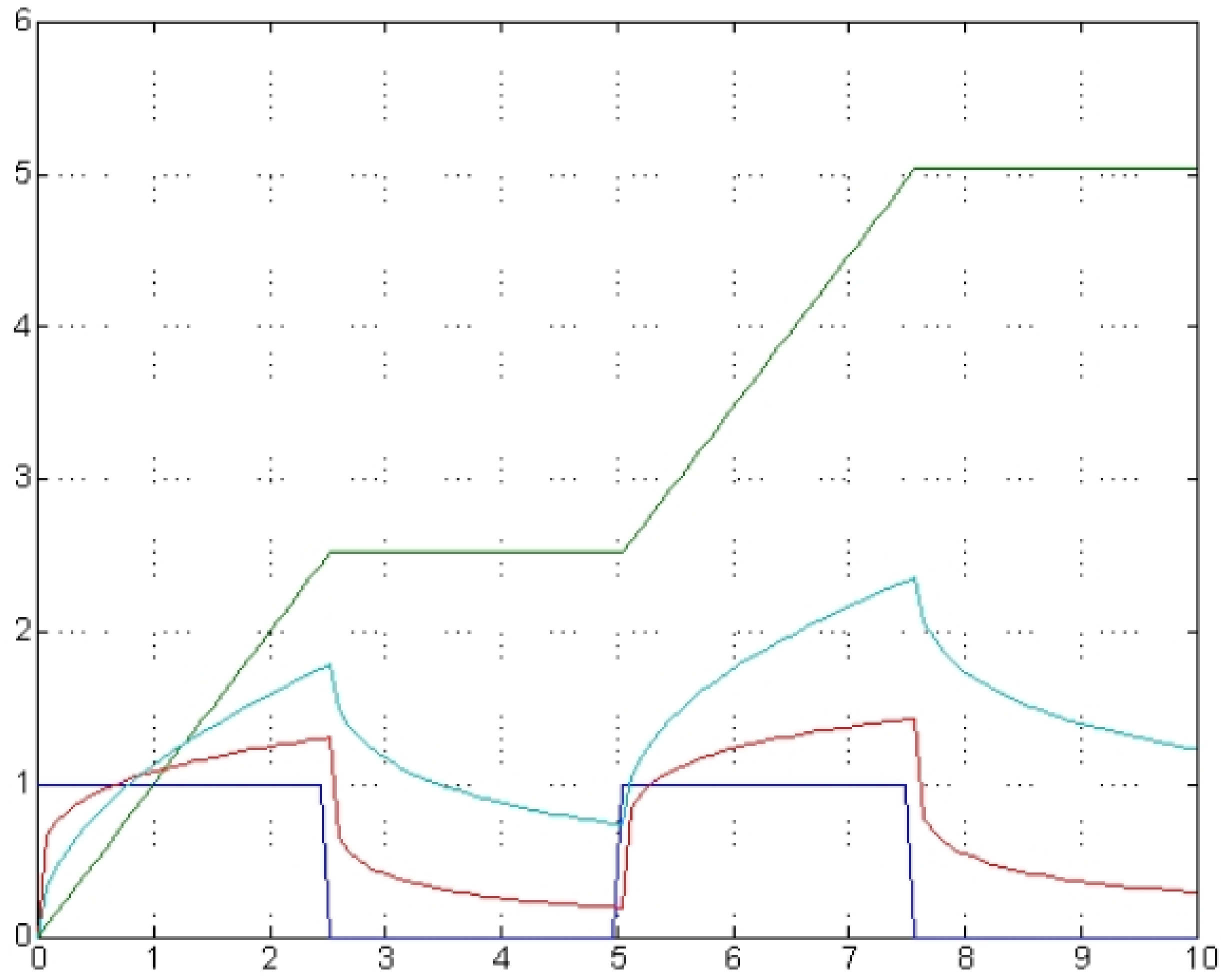
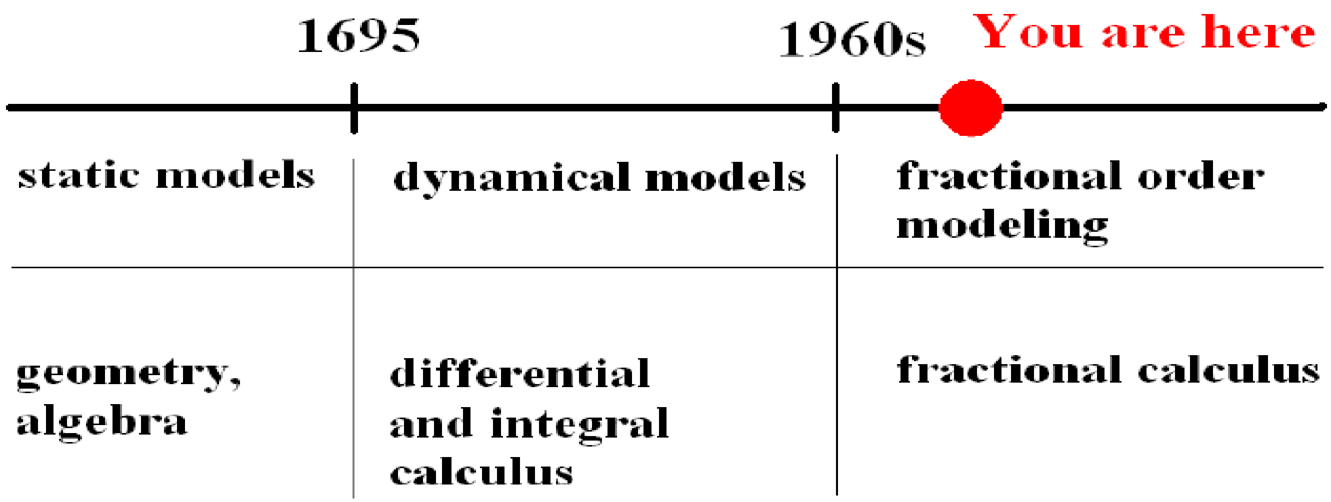
Fractional derivatives of ramp function.



Why and How and When

•Why – Many reasons. Dynamic systems modeling and controls. Better characterization, better control performance
How – Analog versus digital realization methods. Many.
When – **Now**. Ubiquitous. Take a try since we have the new tool. The beginning of a new stage



Slide credit: Igor Podlubny

Modeling: heat transfer

$$\begin{aligned}\frac{\partial^2 y(x,t)}{\partial x^2} &= k^2 \frac{\partial y(x,t)}{\partial t}, \\ & (t>0, \quad 0<x<\infty)\end{aligned}$$

$$\begin{aligned} y(0,t) &= m(t) \\ y(x,0) &= 0 \\ \left|\lim_{x\to\infty} y(x,t)\right| &< \infty \end{aligned}$$

Transfer function:

$$\begin{aligned}\frac{\mathrm{d}^2 Y(x,s)}{\mathrm{d}x^2} &= k^2 s Y(x,s) \\ Q(0,s) &= M(s) \\ \left|\lim_{x\to\infty} Y(x,s)\right| &< \infty \end{aligned}$$

$$Y(x,s) = A(s)e^{-kx\sqrt{s}} + B(s)e^{kx\sqrt{s}}$$

$$\begin{aligned} A(s) &= Y(0,s) = M(s) \\ B(s) &= 0 \end{aligned}$$

$$Y(x,s) = M(s)e^{-kx\sqrt{s}}$$

$$G(s) = \frac{Y(x,s)}{M(s)} = e^{-kx\sqrt{s}}$$

$$\boxed{\textbf{think about transfer function } e^{-\sqrt{s}} \textbf{ !}}$$

FO Controller + IO Plant

Fractional order speed control of DC motor

System transfer function $G(s) = \frac{k}{Js(Ts+1)}$ J being the payload inertia. Phase margin of controlled system:

$$\Phi_m = \arg\left[C(j\omega_g)G(j\omega_g)\right] + \pi$$

Controller: $C(s) = k_1 \frac{k_2 s+1}{s^\alpha}$, $k_2 = T$ giving a **constant phase margin**:

$$\begin{aligned}\Phi_m &= \arg\left[C(j\omega)G(j\omega)\right] + \pi = \arg\left[\frac{k_1 k}{(j\omega)^{(1+\alpha)}}\right] + \pi \\ &= \arg\left[(j\omega)^{-(1+\alpha)}\right] + \pi = \pi - (1+\alpha)\frac{\pi}{2}\end{aligned}$$

Step response:

$$y(t) = \pounds^{-1}\left\{\frac{kk_1/J}{s\left(s^{1+\alpha} + kk_1/J\right)}\right\} = \left(\frac{kk_1}{J}\right) t^{1+\alpha} E_{1+\alpha,2+\alpha}\left(-\frac{kk_1}{J} t^{1+\alpha}\right) \quad (63)$$

Mittag-Leffler function: definition

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad (\alpha > 0, \quad \beta > 0)$$

$$E_{1,1}(z) = e^z,$$

$$E_{2,1}(z^2) = \cosh(z), \quad E_{2,2}(z^2) = \frac{\sinh(z)}{z}.$$

$$E_{1/2,1}(z) = e^{z^2}\mathrm{erfc}(-z);$$

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt.$$

Slide credit: Igor Podlubny

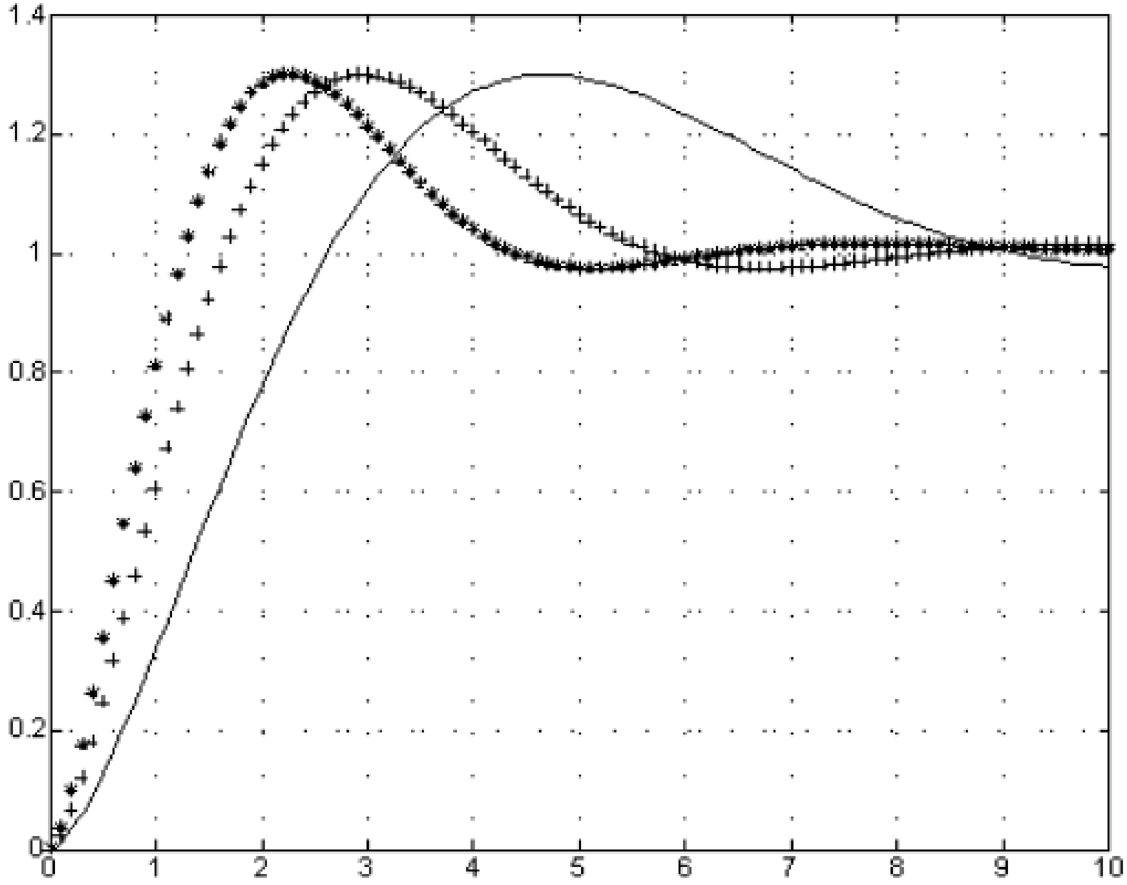
G. M. Mittag-Leffler

Professor Donald E. Knuth, creator of TEX:

"As far as the spacing in mathematics is concerned... I took *Acta Mathematica*, from 1910 approximately; this was a journal in Sweden ... Mittag-Leffler was the editor, and his wife was very rich, and they had the highest budget for making quality mathematics printing. So the typography was especially good in *Acta Mathematica*."

(Questions and Answers with Prof. Donald E. Knuth, Charles University, Prague, March 1996)

Slide credit: Igor Podlubny



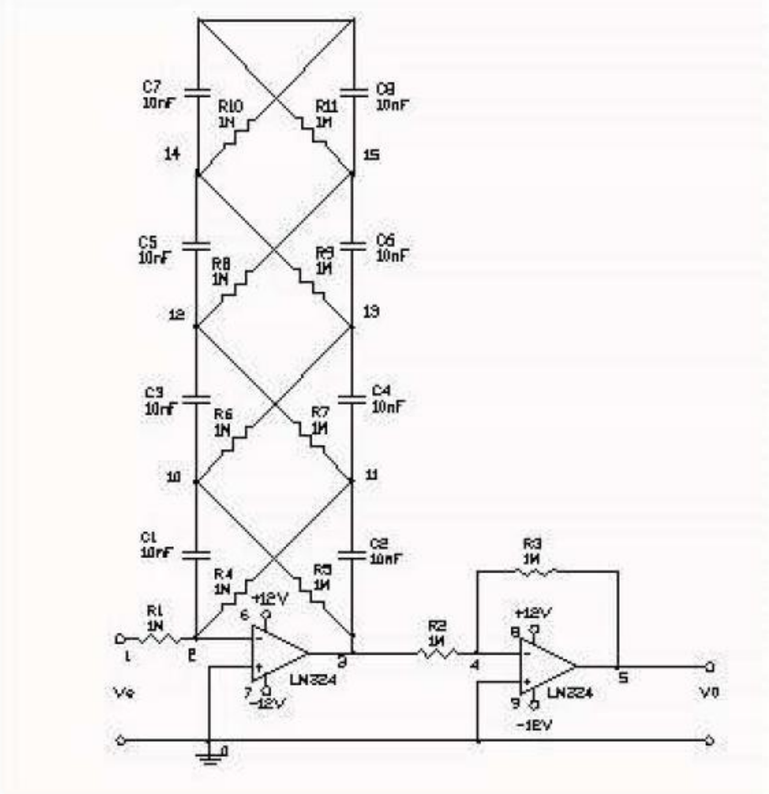
Note the iso-damping (similar overshoot!)

Fractional operator

- First order differentiator: s
- First order integrator: $1/s$

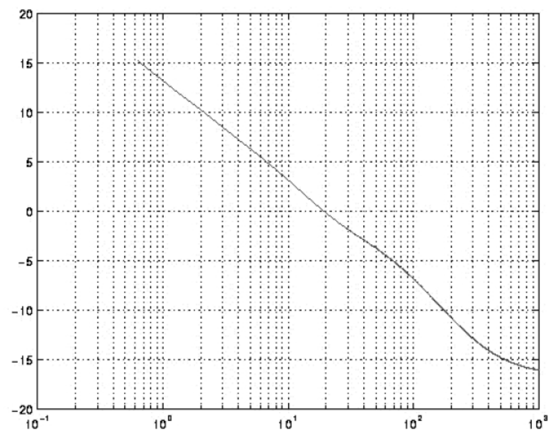
What is s^{α} when α is a *non-integer*?

Possible? Possible!

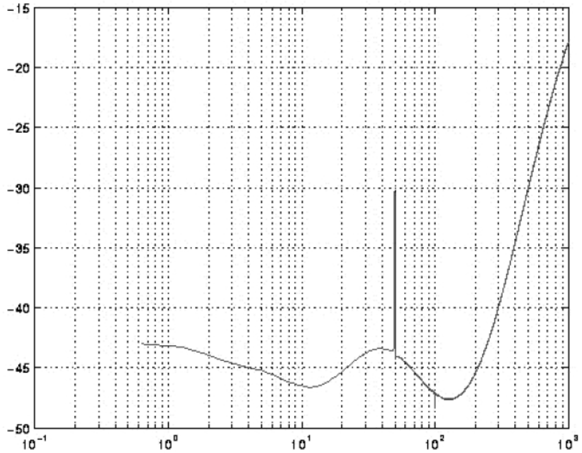


Analog $1/\sqrt{s}$ using op-amps.

Magnitude plot (dB vs. rad./sec.)



Phase plot (deg. vs. rad./sec.



I. Petras, I. Podlubny, P. O'Leary, L. Dorcak, and Vinagre B. "**Analogue Realization of Fractional Order Controllers**". FBERG, Technical University of Kosice, Kosice, Slovak, **ISBN 8070996277** edition, 2002.

Fractor: Analogue device

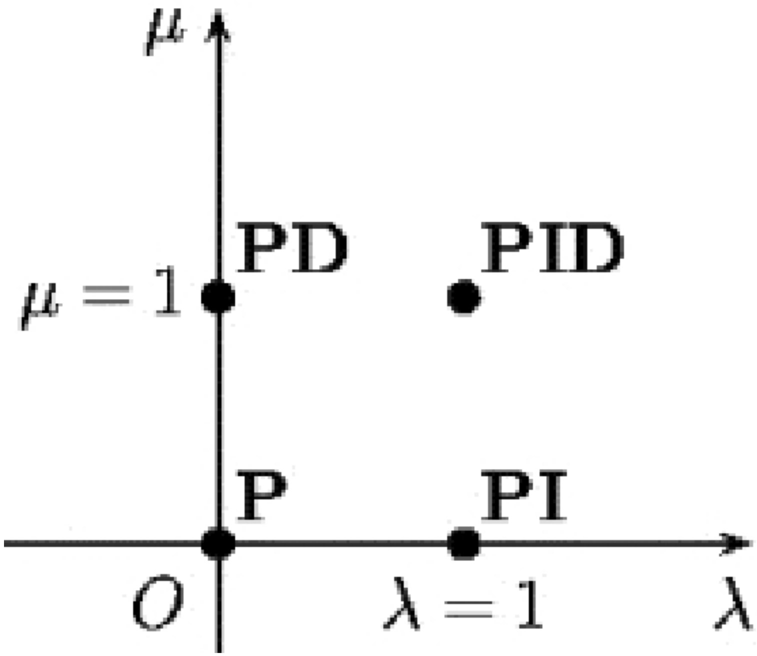
Fractional Calculus Day at USU, April 19, 2005

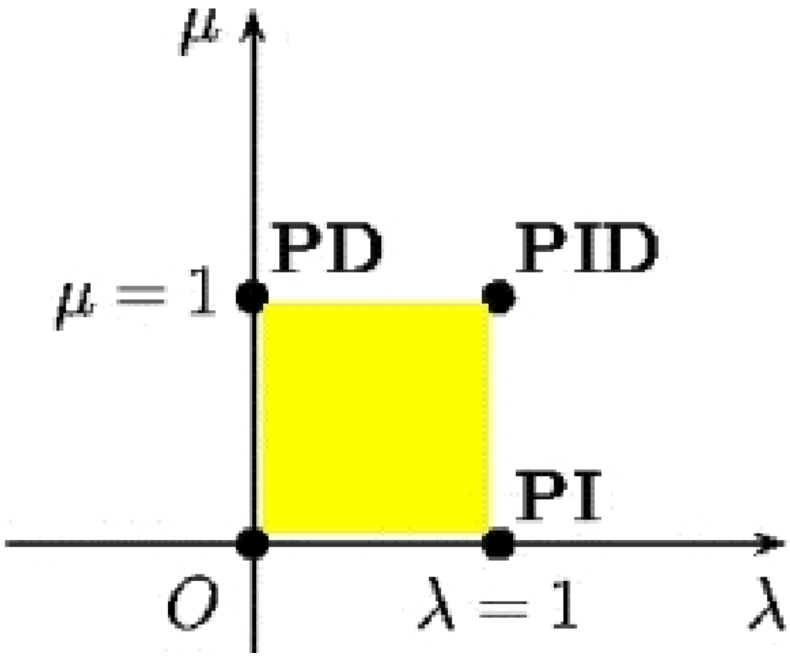
Photo credit: Igor Podlubny

Fractional order PID control

- 90% are PI/PID type in industry. (**Ubiquitous**)

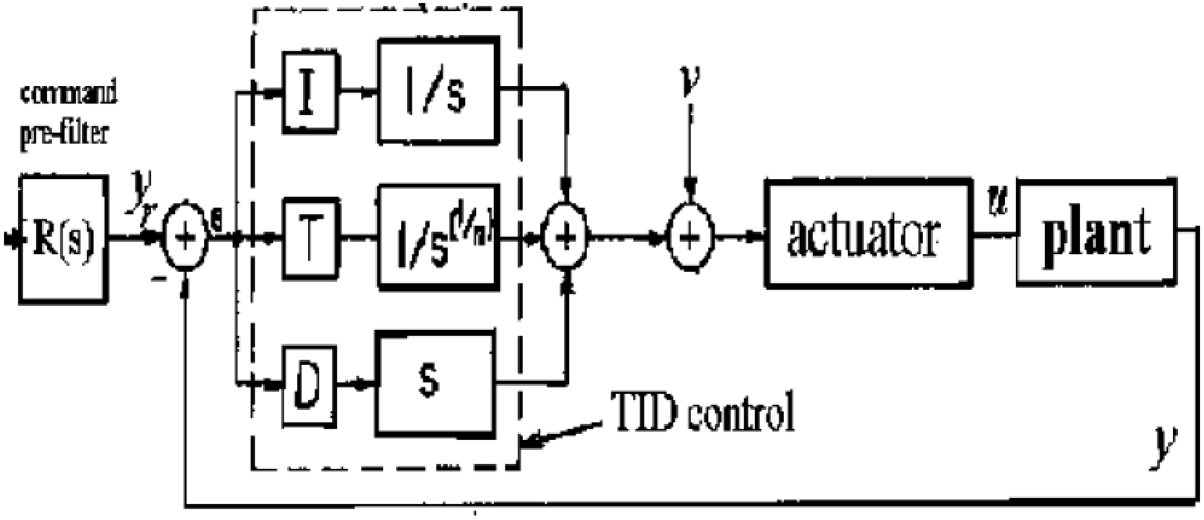
$$u(t) = K_p(e(t) + T_i \mathrm{D}_t^{-\lambda} e(t) + \frac{1}{T_d} \mathrm{D}_t^{\mu} e(t)). \qquad (\mathrm{D}_t^{(*)} \equiv {}_0\mathrm{D}_t^{(*)}).$$





Igor Podlubny. "*Fractional-order systems and* $PI^{\lambda}D^{\mu}$*-controllers*". *IEEE Trans. Automatic Control, 44(1): 208–214, 1999.*

*YangQuan Chen, Dingyu Xue, and Huifang Dou. "**Fractional Calculus and Biomimetic Control**". IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shengyang, China.*

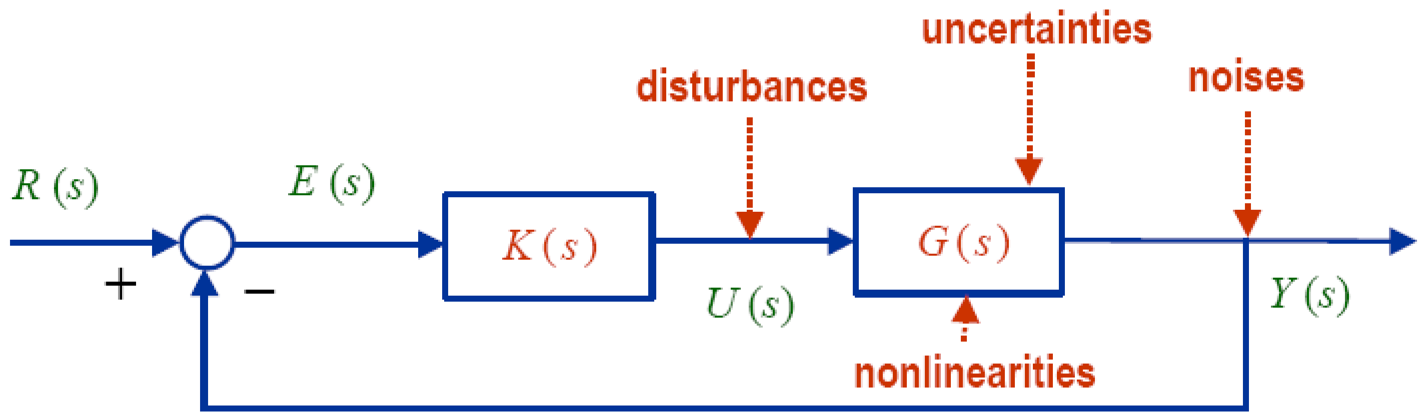


US05371670 on TID by B. J. Lurie, 1994

"3-param. tunable tilt-integral-deriv. controller"

Fractional Order Controls

- IO Controller + IO Plant
- FO Controller + IO Plant
- FO Controller + FO Plant
- IO Controller + FO Plant



D. Xue and **Y. Chen***, "A Comparative Introduction of Four Fractional Order Controllers". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

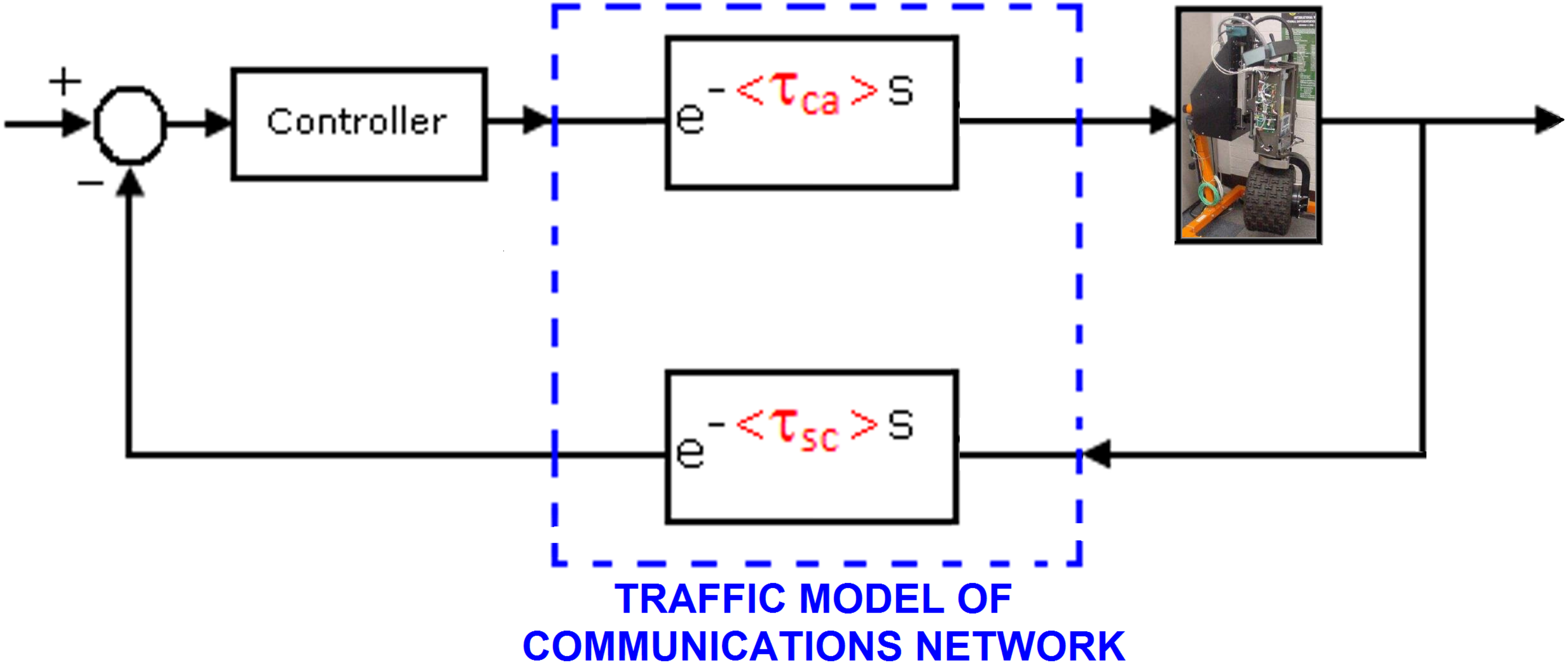
Think "fractional order"

- **Fractional-order Systems and Controls**
- Fundamentals and Applications
- Series: Advances in Industrial Control
- **Monje**, C.A., **Chen**, Y., **Vinagre**, B.M., **Xue**, D., **Feliu**, V.
- 1st Edition., 2010, XXII, 418 p. 100 illus. With online files/update., Hardcover
- ISBN: 978-1-84996-334-3

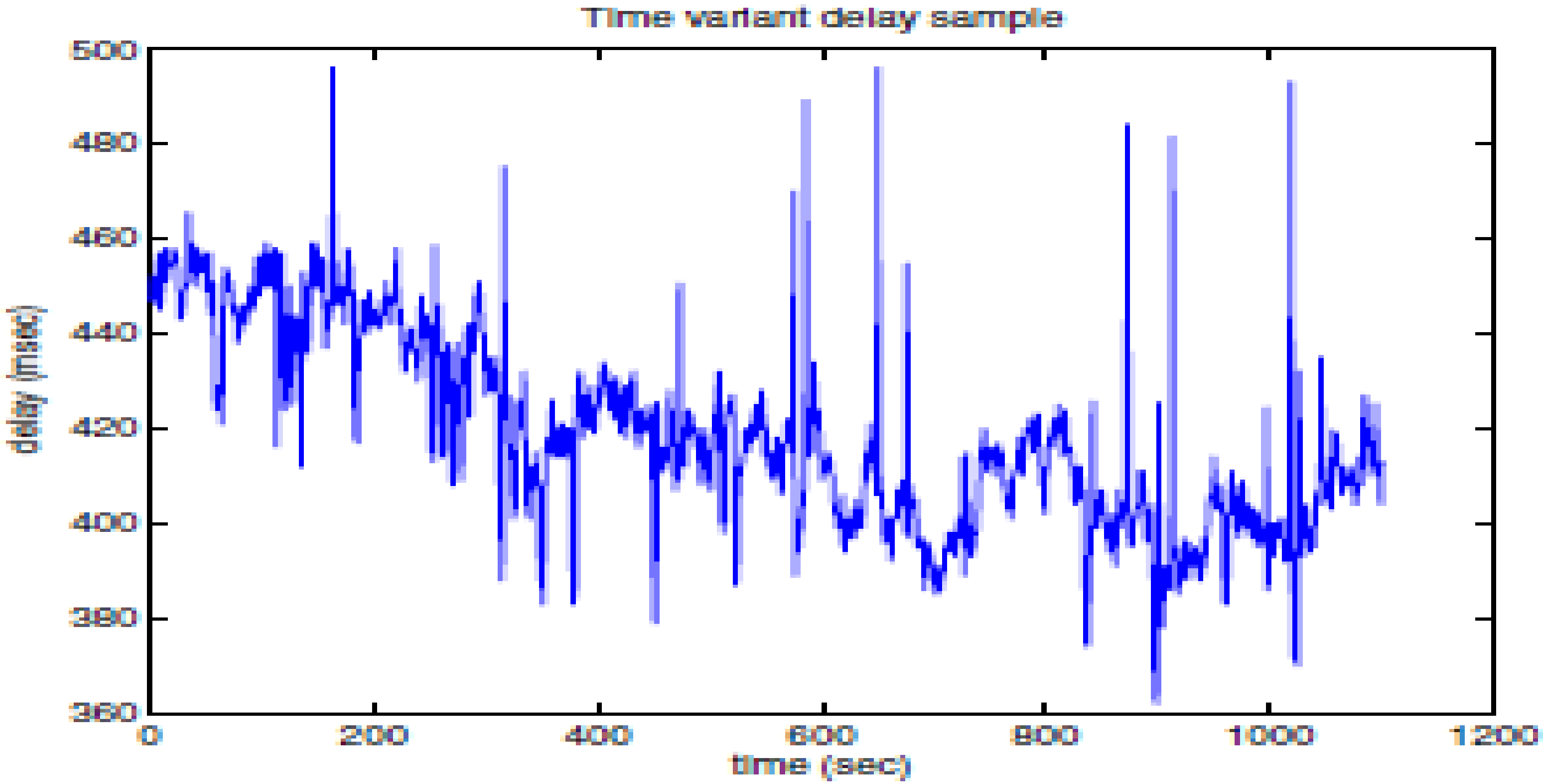
Outline

- CSOIS (Center for Self-Organizing & Intelligent Systems)
- Fractional Calculus and Fractional Order Thinking
- Delay Dynamics
- Networked Control Systems
- Concluding Remarks

NCS – delay is random, time-varying

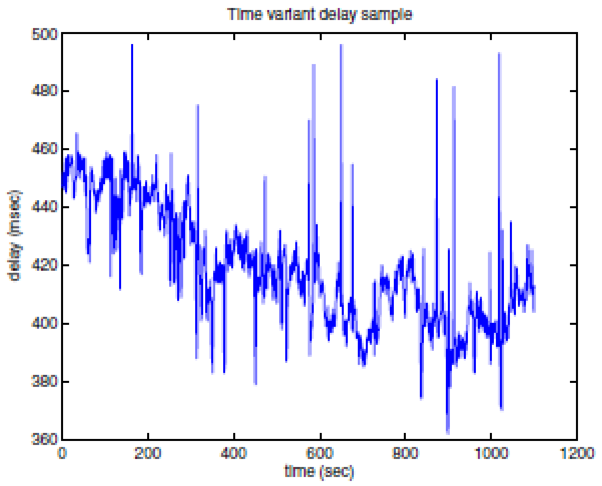


… and spiky

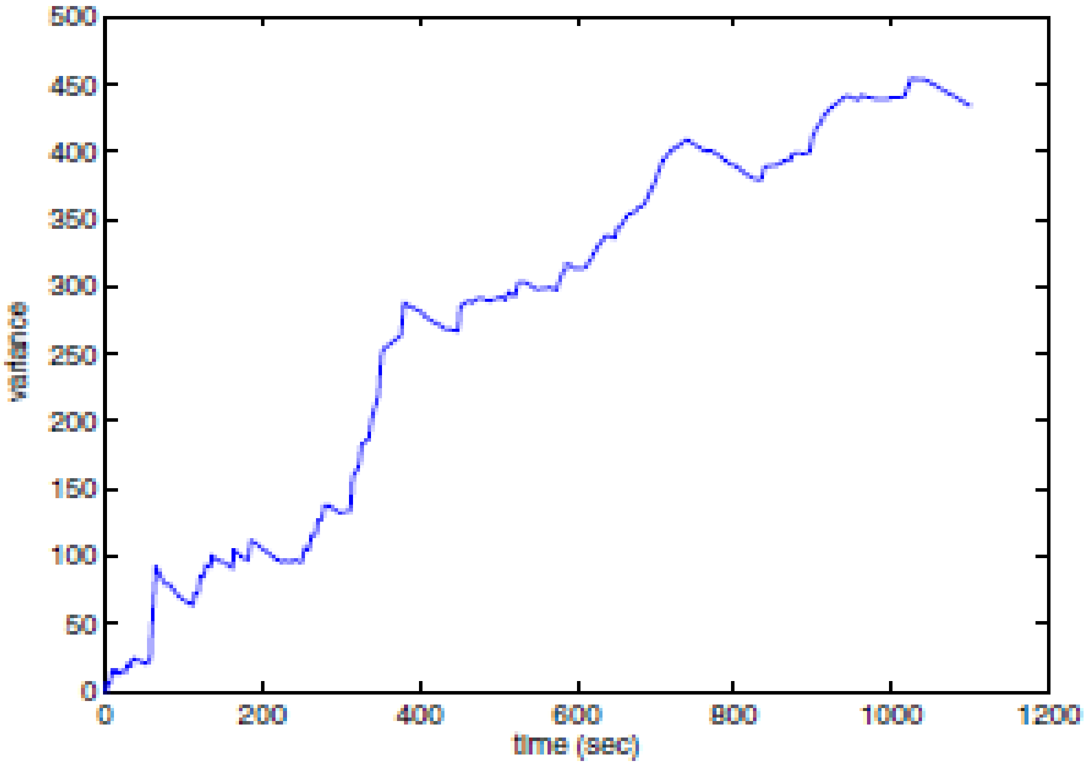


(a) Network delay samples

PROBLEM? running variance estimate is not convergent

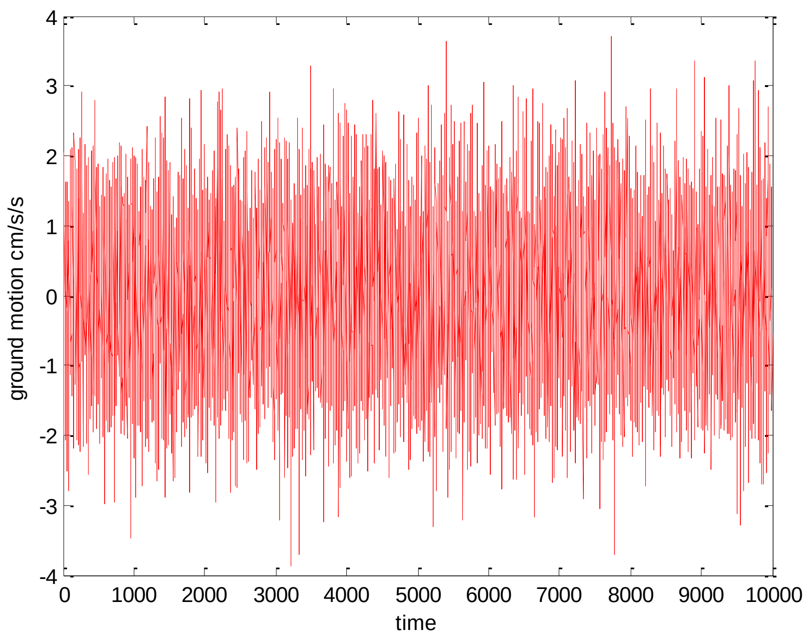


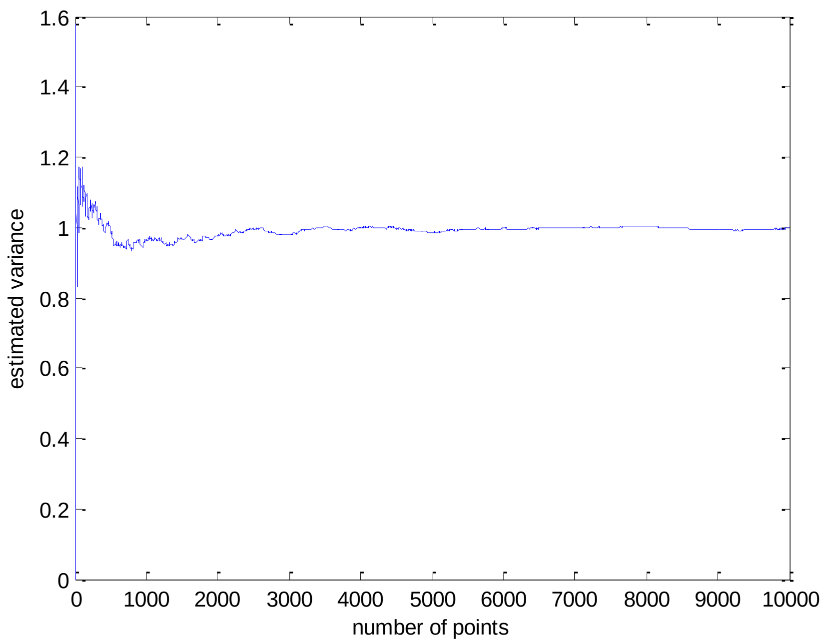
(a) Network delay samples



(b) infinite or divergent variance

Noise - 1

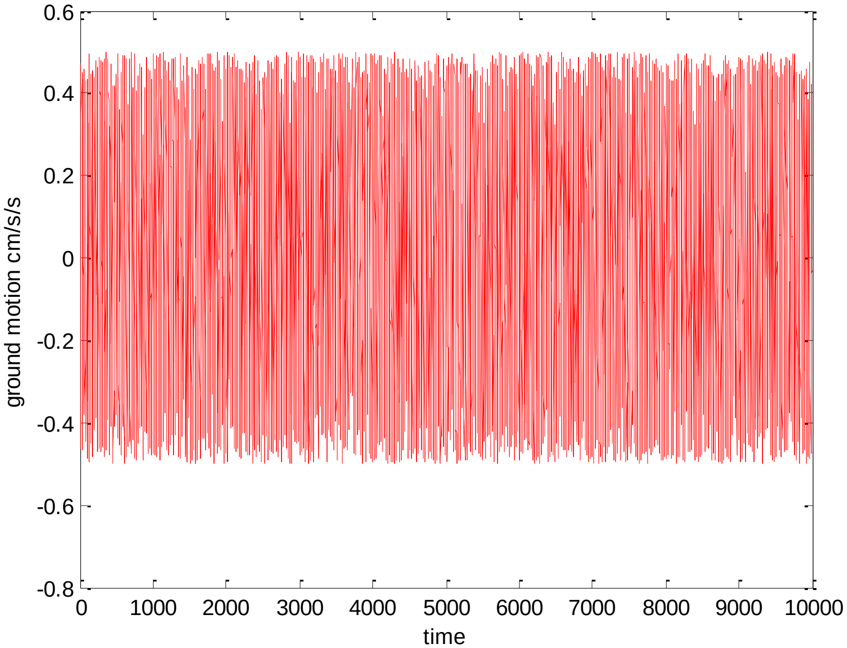


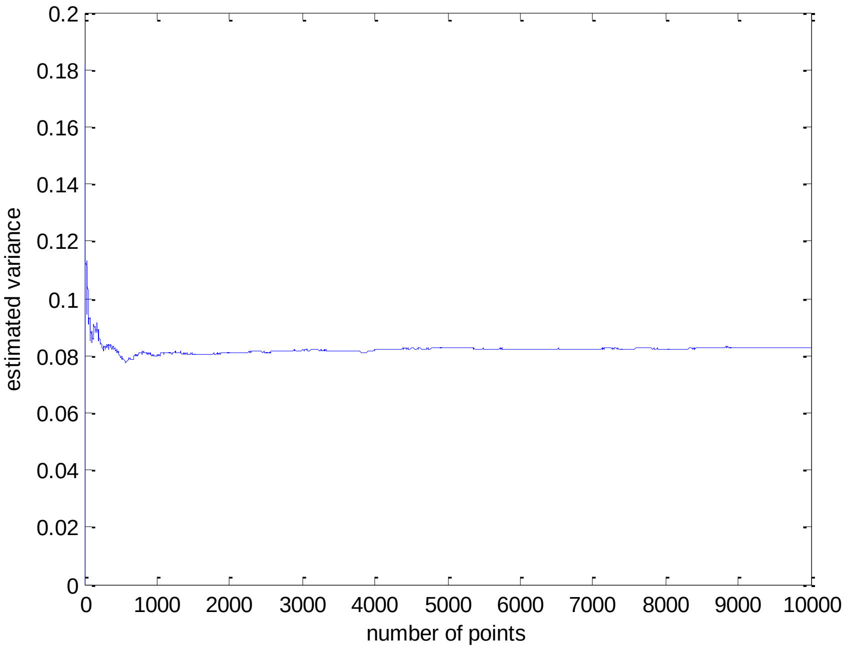


Normal distribution N(0,1)

Sample Variance

Noise - 2





Uniformly distributed

Sample Variance

Fractional Lower Order Statistics (FLOS) or Fractional Lower Order Moments (FLOM)

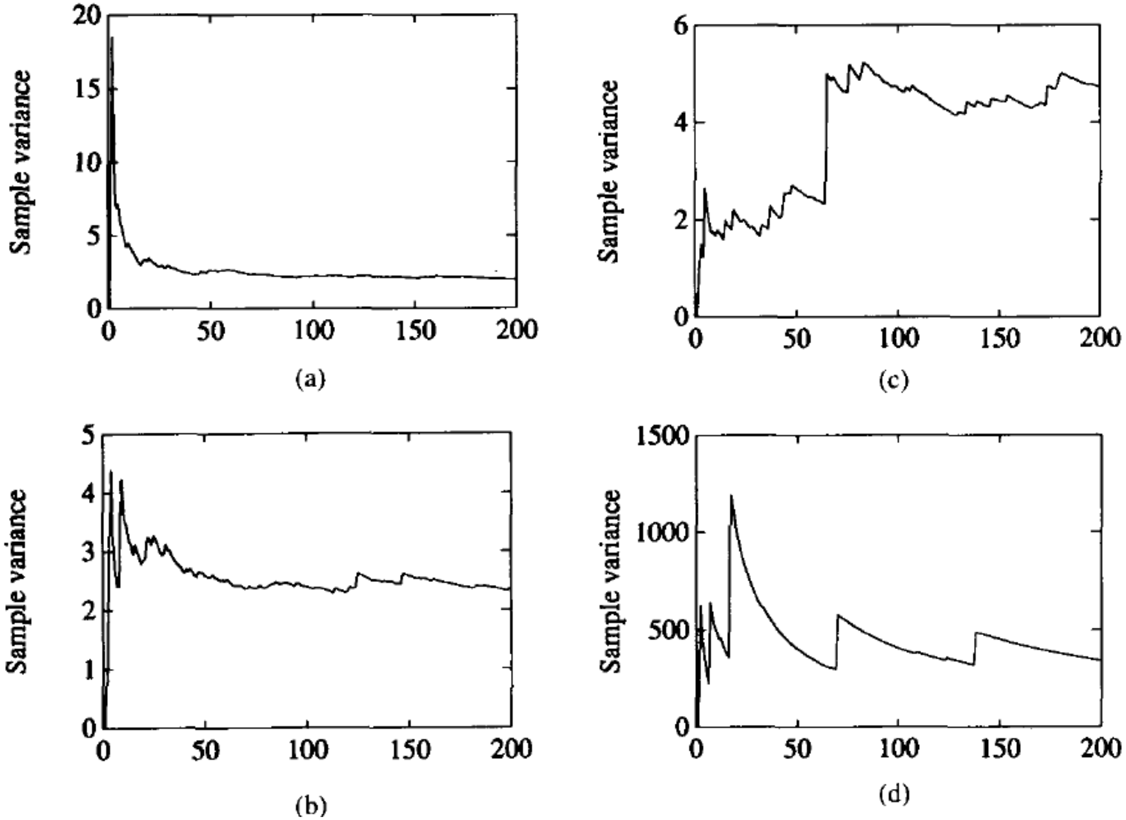


Fig. 2. Running sample variances for four different values of α: (a) $\alpha = 2.0$; (b) $\alpha = 1.9$; (c) $\alpha = 1.5$; (d) $\alpha = 1.1$.

Shao, M., and Nikias, C. L., 1993. "Signal processing with fractional lower order moments: stable processes and their applications". Proceedings of the IEEE, 81 (7) , pp. 986 – 1010.

Important Remarks

A simple test of infinite variance is to plot the running sample variance estimate S_n with respect to number of points n where $S_n^2 = (\sum_{k=1}^{n}(x_k - \bar{x}_n)^2)/(n-1)$ and $\bar{x}_n = \sum_{k=1}^{n} x_k/n$. For finite variance processes x_k, S_n will converge to a constant value as n increases. If S_n does not converge to a constant value, x_k is a non-Gaussian infinite-variance process with fractional lower order $\alpha < 2$.

In fact, for a non-Gaussian stable distribution with characteristic exponent α, only the moments of orders less than α are finite. Therefore, variance can no longer be used as a measure of dispersion and in turn, many standard signal processing techniques such as spectral analysis and all least squares (LS) based methods may give misleading results.

Long-range dependence

- History: The first model for long range dependence was introduced by Mandelbrot and Van Ness (1968)
- Value: financial data

  communications networks data

  video traffic

  biocorrosion data

Long-range dependence

- Consider a second order stationary time series $Y = \{Y(k)\}$ with mean zero. The time series Y is said to be long-range dependent if

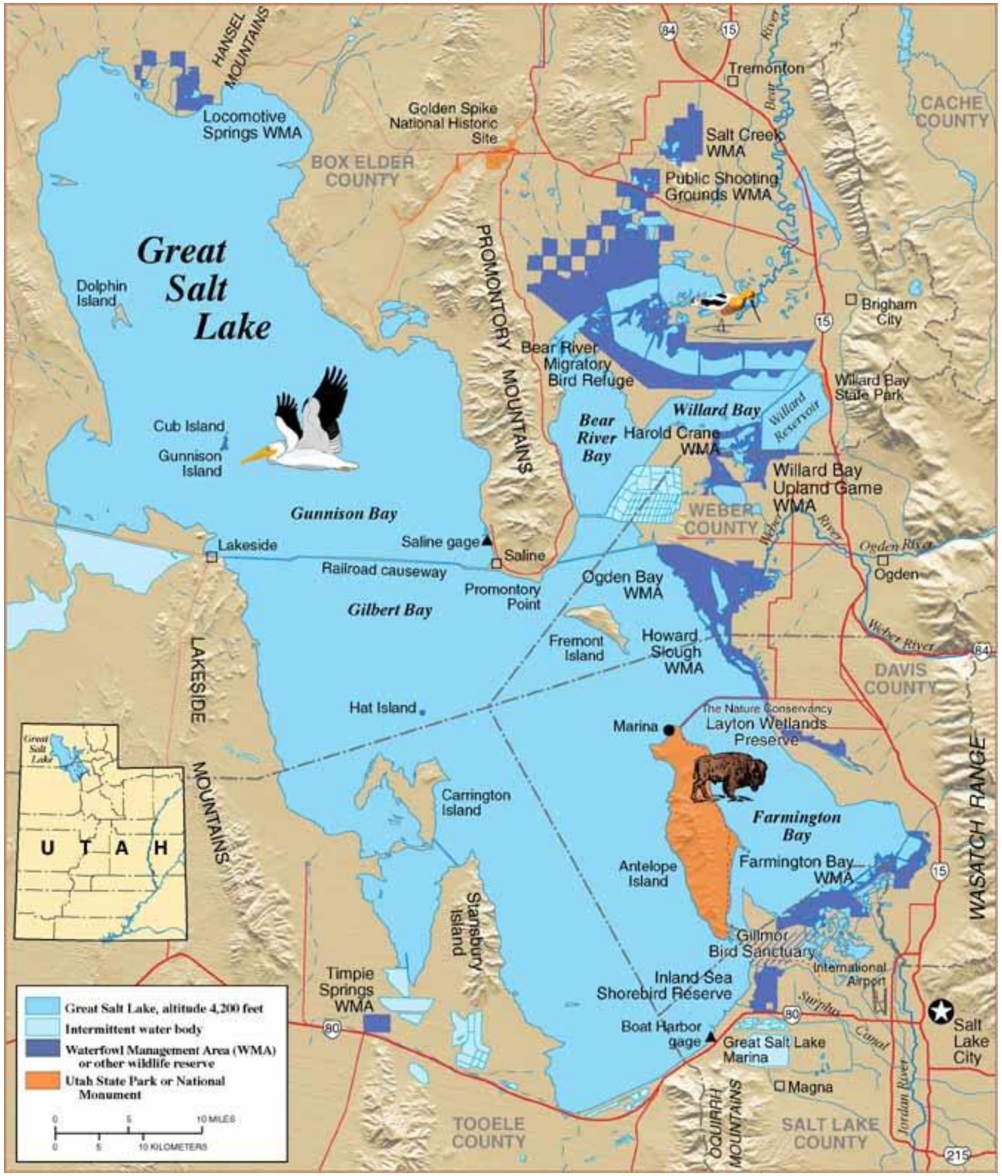
$$r_Y(k) = EY(k)Y(0) \sim c_Y |k|^{-\gamma}, k \to \infty, 0 < \gamma < 1$$

$$s_Y(\xi) \sim c_s |\xi|^{-\alpha}, 0 < \alpha < 1,$$

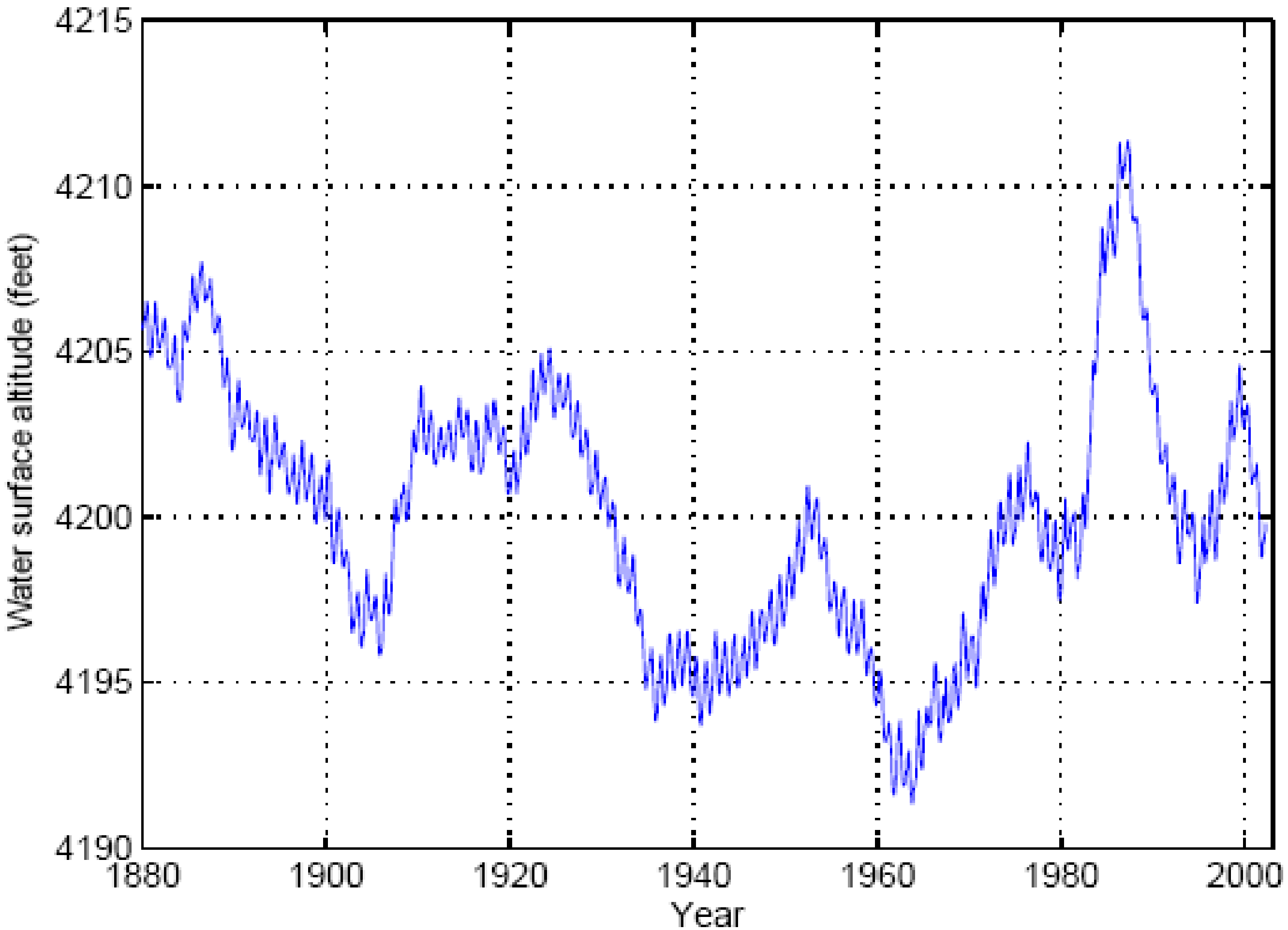
Utah State University

GSL: Do you care about it?

CSOIS
CENTER FOR SELF-ORGANIZING AND INTELLIGENT SYSTEMS



Long-term water-surface elevation graphs of the Great Salt Lake



Elevation Records of Great Salt Lake

- The Great Salt Lake, located in Utah, U.S.A, is the fourth largest terminal lake in the world with drainage area of 90,000 km^2.
- The United States Geological Survey (USGS) has been collecting water-surface-elevation data from Great Salt Lake since 1875.
- The modern era record-breaking rise of GSL level between 1982 and 1986 resulted in severe economic impact. The lake levels rose to a new historic high level of 4211:85 ft in 1986, 12.2 ft of this increase occurring after 1982.
- The rise in the lake since 1982 had caused 285 million U.S. dollars worth of damage to lakeside.
- According to the research in recent years, traditional time series analysis methods and models were found to be insufficient to describe adequately this dramatic rise and fall of GSL levels.
- This opened up the possibility of investigating whether there is long-range dependence in GSL water-surface-elevation data so that we can apply FOSP to it.

A recent paper

- *"FARIMA with stable innovations model of Great Salt Lake elevation time series"*
  - Hu Sheng and YangQuan Chen.
  - Signal Processing, 2010 (in press)
  - doi:10.1016/j.sigpro.2010.01.023

Two older papers

- Rongtao Sun+ and YangQuan Chen* and Qianru Li+. “Modeling and Prediction of Great Salt Lake Elevation Time Series Based on ARFIMA”. DETC2007-34905 in *Proc. of the ASME Design Engineering Technical Conferences*, Sept. 4-7, 2007 Las Vegas, NE, USA, 3rd ASME Symposium on Fractional Derivatives and Their Applications (FDTA'07), part of the 6th ASME International Conference on Multibody Systems, Nonlinear Dynamics, and Control (MSNDC). 11 pages.

- Qianru Li+ and Christophe Tricaud+ and YangQuan Chen*. “Great Salk Lake Level Forecasting Using FIGARCH (FIG Autoregressive conditional heteroskedasticity) Model” DETC2007-34909 in *Proc. of the ASME Design Engineering Technical Conferences*, Sept. 4-7, 2007 Las Vegas, NE, USA, 3rd ASME Symposium on Fractional Derivatives and Their Applications (FDTA'07), part of the 6th ASME International Conference on Multibody Systems, Nonlinear Dynamics, and Control (MSNDC). 10 pages.

FOSP Techniques

- Fractional derivative and integral
- Fractional linear system
- Autoregressive fractional integral moving average
- 1/f noise
- Hurst parameter estimation
- Fractional Fourier Transform
- Fractional Cosine, Sine and Hartley transform
- Fractals
- Fractional Splines
- Fractional Lower Order Moments (FLOM) and Fractional Lower Order Statistics (FLOS)

Fractional Calculus, LRD, Power Law,

$u(t)$

White Noise

$$H(s)=\frac{1}{s^{\alpha}}$$

$y(t)$

$$h(t)=\frac{t^{\alpha-1}}{\Gamma(\alpha)}$$

$$Ryy(\tau)=\sigma^{2}\frac{|\tau|^{2\alpha-1}}{2\Gamma(2\alpha)\cos\alpha\pi}$$

y(t) is a Brownian motion when α=1, i.e., $1/f^{2}$ process.

$1/f^{2\alpha}$ noise (signal) generation via fractional dynamic system

Power laws in

- Signal/Systems
- Probability distribution
- Random processes (correlation functions)

Rule of thumb for Fractional Order Thinking

- Self-similar
- Scale-free/Scale-invariant
- Power law
- Long range dependence (LRD)
- $1/f^{a}$ noise
- Porous media
- Particulate
- Granular
- Lossy
- Anomaly
- Disorder
- Soil, tissue, electrodes, bio, nano, network, transport, diffusion, soft matters (**bio**x) …

Power law and power law Lyapunov

- "Power law is ubiquitous" – John Doyle 2001 IEEE CDC Plenary Talk http://www.cds.caltech.edu/~doyle/CDC2001/index.htm
- "When you talk about power law, you are talking actually about fractional order calculus!" – YangQuan Chen 2006 IFAC FDA06 Plenary Talk
- "Lyapunov is ubiquitous in control literature" – ibid.

Intuitions

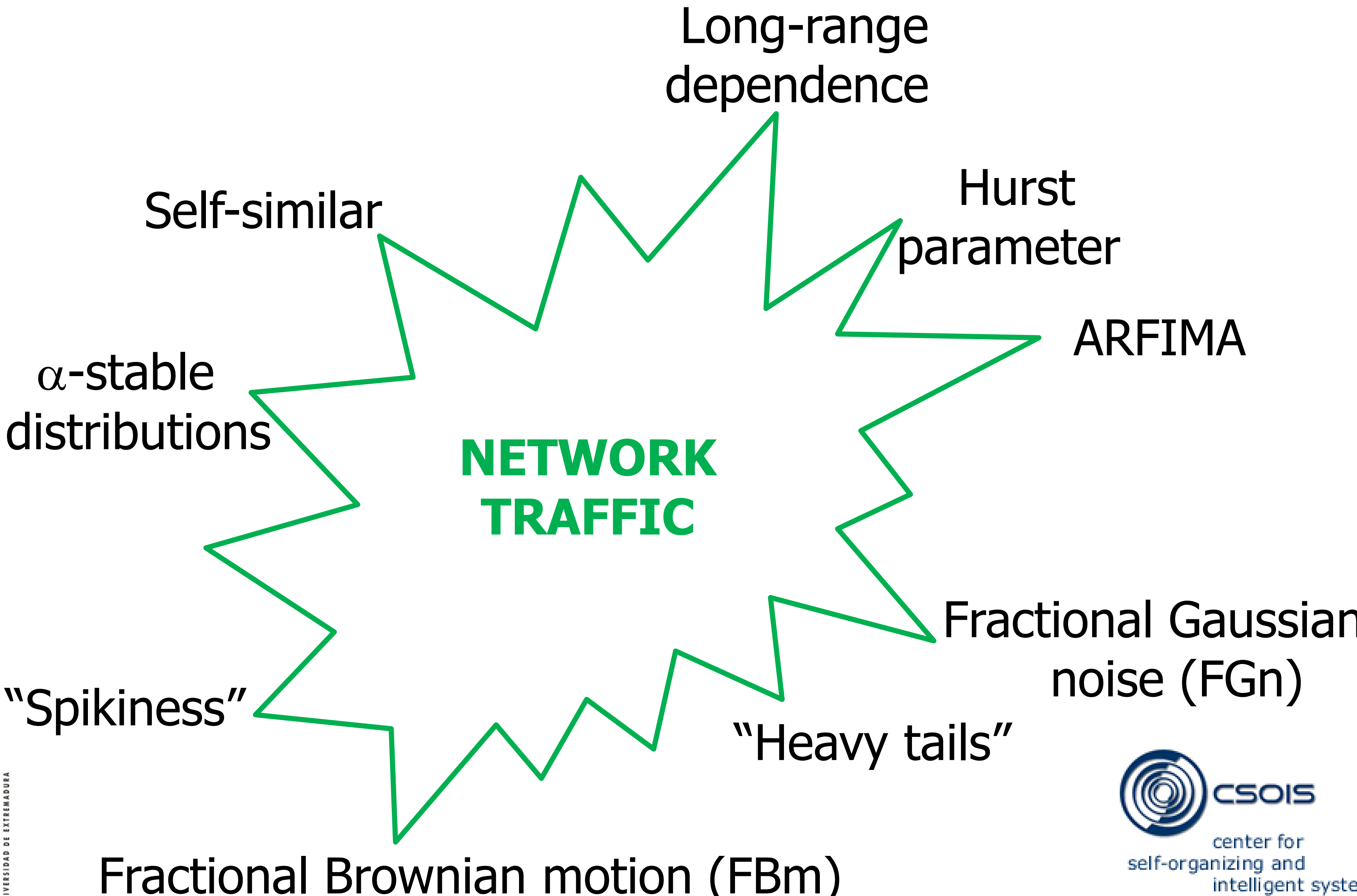
- One does not have to be rich to be smart.
- One does not have to be smart to use fractional order calculus.
- **A dynamic system does not have to make the "*generalized energy*" decay exponentially to be stable!**

Y. Li, Y. Q. Chen and I. Podlubny. "Mittag-Leffler Stability of Fractional Order Nonlinear Systems", *Automatica*, 45(8): 965-1969, 2009. DOI: 10.1016/j.automatica.2009.04.003

Applications – C-FOSE Proposal (Center for Fractional Order Systems Engineering)

1. Human-augmentation
2. Human Nerve System
3. Robotic equipment
4. Electric drive systems
5. Power Converters
6. Disk drive servo
7. Audio signal processing
8. Aircraft
9. Automobiles
10. Fuel cells
11. Lidar, radar, sonar, ultrasonic imaging
12. Battery chargers
13. Nuclear reactors
14. Temperature Control
15. Biosensor signal processing

CONCEPTS RELATED TO



MODELS IN LITERATURE (I)

NETWORK TRAFFIC MODEL

Fractional Brownian motion (FBm)

[1] O.I. Sheludin, S.M. Smolskiy, and A.V. Osin, *Self-Similar Processes in Telecommunications*. John Wile & Sons, Ltd, England, 2007.

α-stable distributions

[10] S. Mukhopadhyay, Y. Han, and Y.Q. Chen, "Fractional Order Networked Control Systems and Random Delay Dynamics: a Hardware-in-the-Loop Simulation Study". In: *Proceedings of the 2009 American Control Conference*, pp. 1418-1423, USA, 2009.

[11] W. Qin, Q. Wang, and A. Sivasubramiam, "An α-stable Model-based Linear-parameter-varying Control for Managing Server Performance Under Self-similar Workloads". *IEEE Transactions on Control Systems Technology*, Vol. 17, No. 1, pp. 123-134, January 2009.

Fractional Autorregresive Moving Average (ARFIMA) proccess

[7] S. Stoev, and M.S. Taqqu, "Simulation Methods for Linear Fractional Stable Motion and FARIMA Using the Fast Fourier Transform". *Fractals*. Vol. 12, No. 1, pp. 95-121, 2004.

[27] A. Scherrer, N. Larrieu, P. Owerzarski, P. Borgnat, and P. Abry, "Non-Gaussian and Long Memory Statistical Characterisations for Internet Traffic with Anomalies". *IEEE Transactions on Dependable and Secure Computing*, Vol. 4, No. 1, pp. 56-70, 2007.

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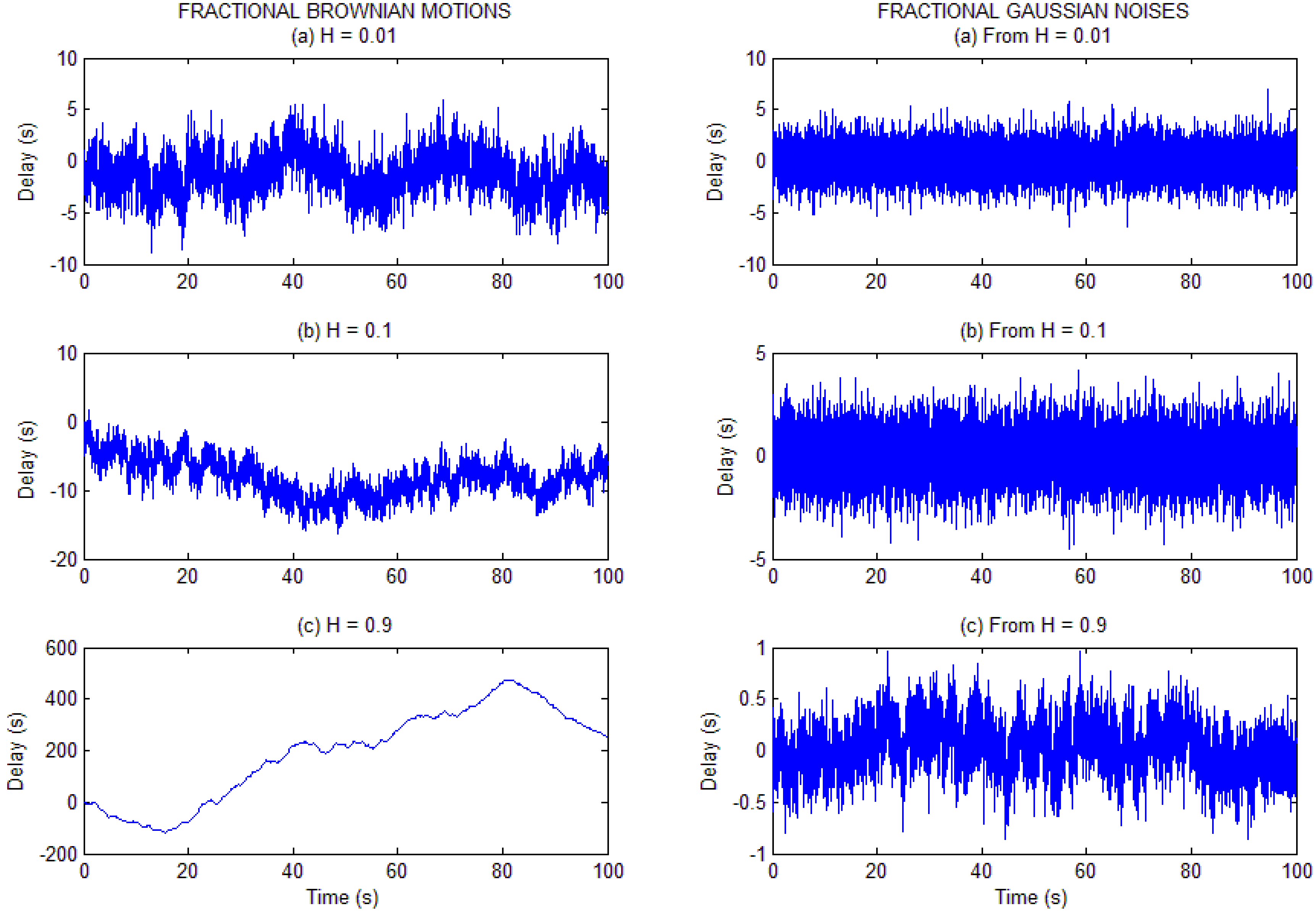
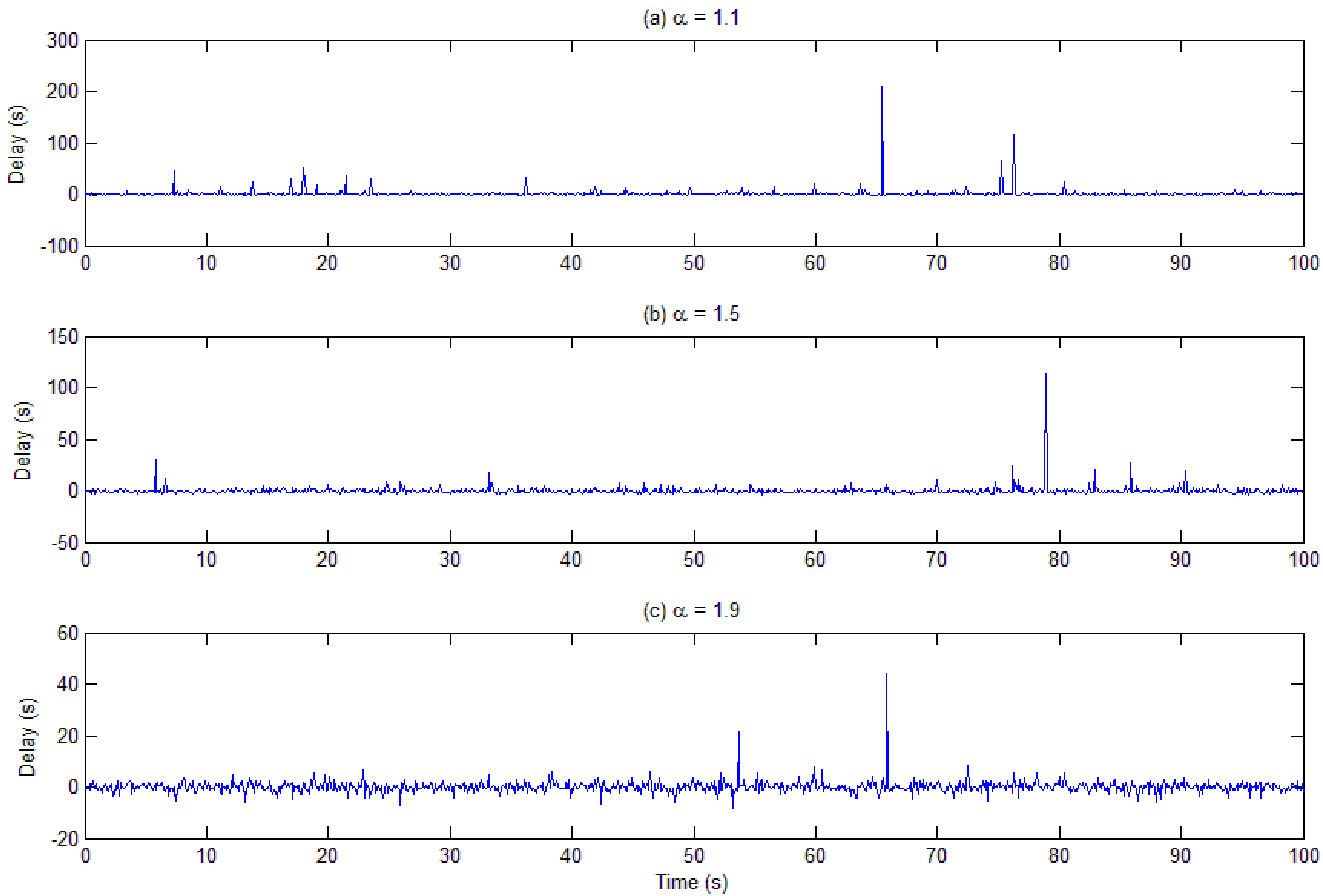
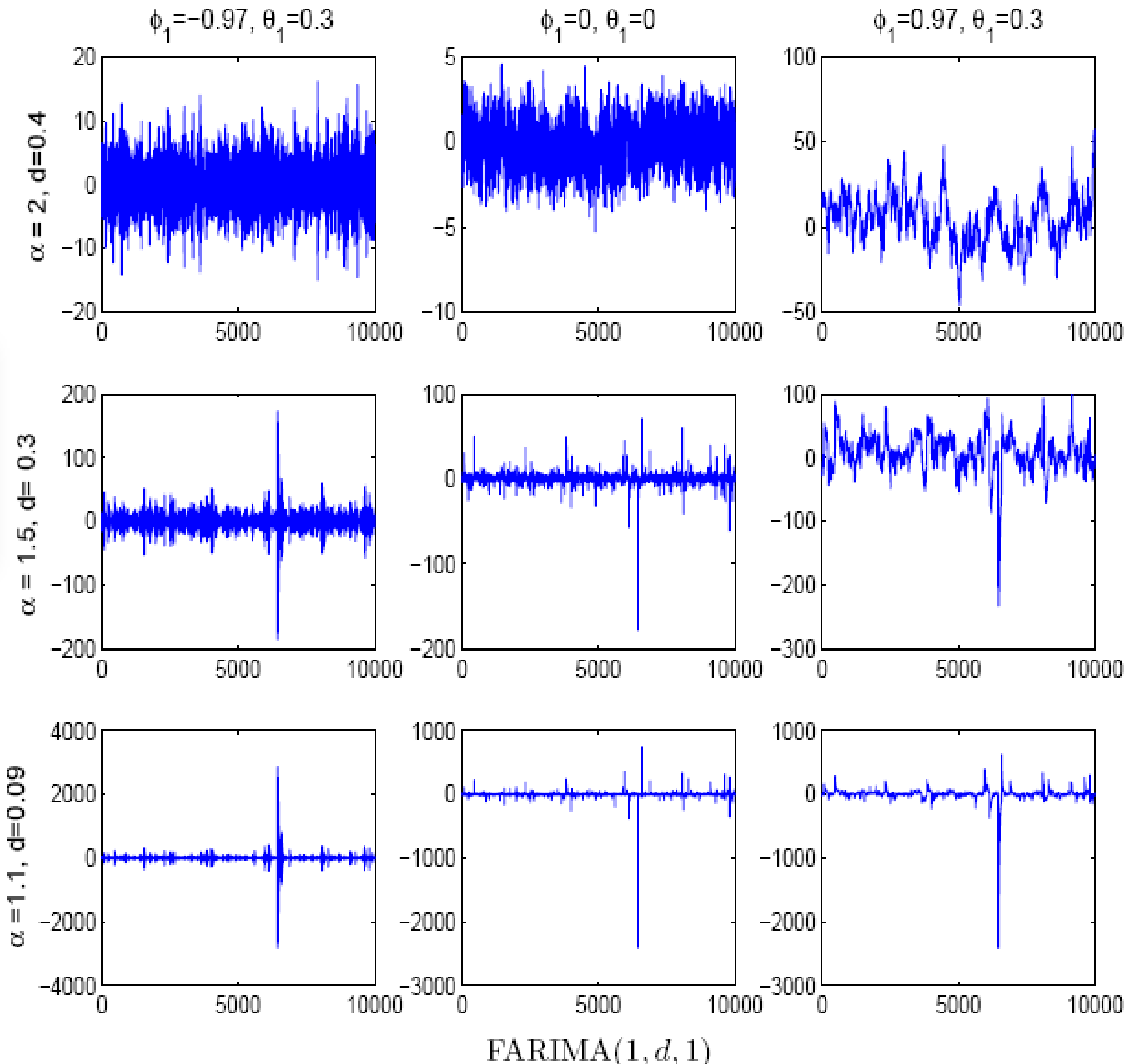
MODELS IN LITERATURE (II)



MODELS IN LITERATURE (III)



MODELS IN LITERATURE (&IV)



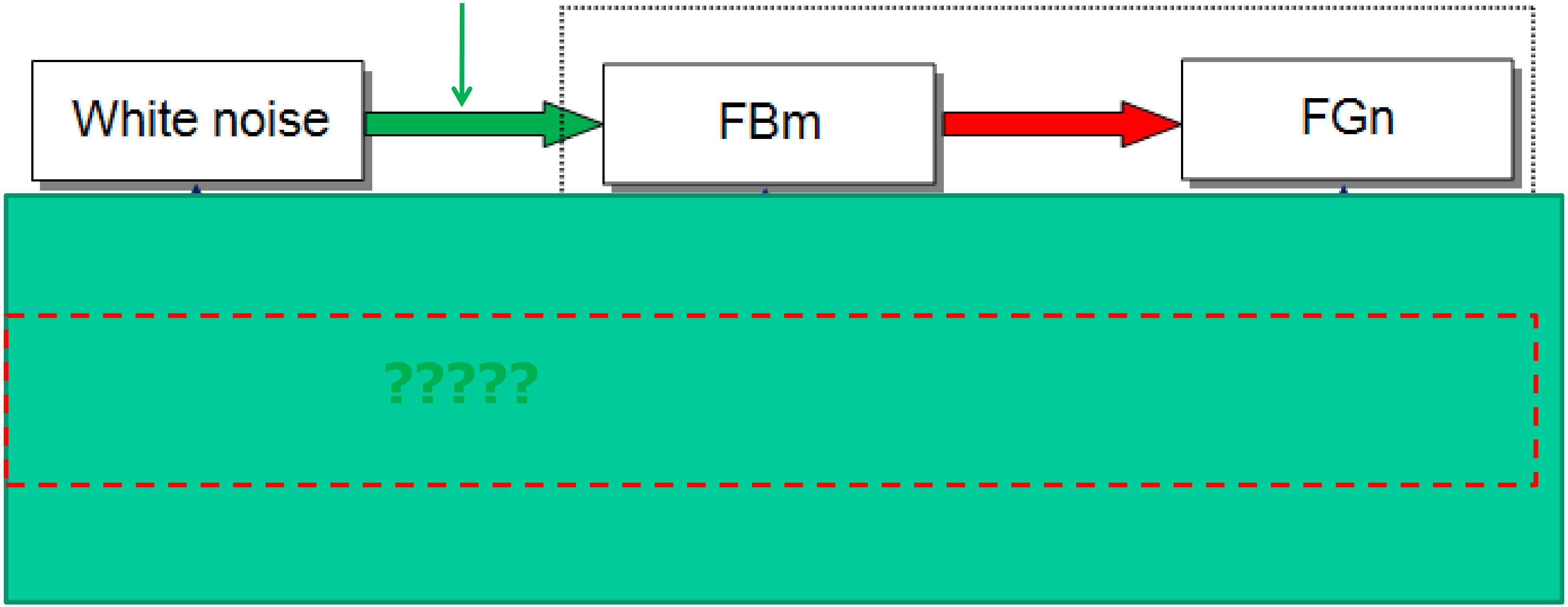
[7] S. Stoev, and M.S. Taqqu, "Simulation Methods for Linear Fractional Stable Motion and FARIMA Using the Fast Fourier Transform". *Fractals*, 2004.

FO NETWORK DELAY DYNAMICS

$$\boxed{D^{\beta}\tau(t) = B(t)}$$

β the fractional-order,
$\tau(t)$ the network-induced delay,
$B(t)$ white noise.

[18] V. Pipiras, and M.S Taqqu, "Fractional calculus and its connections to fractional Brownian motion". *Theory and Applications of Long-range Dependence*, 2003.



- Self-similarity => Hurst parameter
- "Spikiness".

Operation: $D^{-\beta}$

Operation: *increments*

Self-similar processes

Outline

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Back to control

Networked Control System:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t - \tau(t)), \quad x(0) = x_0$$

$$y(t) = (C + \Delta C)x(t)$$

and Delay Dynamics:

$$\frac{\mathrm{d}^\alpha z(t)}{\mathrm{d}t^\alpha} = f(z(t), \epsilon_0(t), t)$$

$$\tau(t) = h(z(t), \epsilon_0(t), t)$$

$$u(t) = -K_1 x(t) = -K_1 [e^{A\tau_1(t)} x(t - \tau_1(t))$$

$$+ e^{At} \int_{t-\tau_1(t)}^{t} e^{-A\xi} B u(\xi) \mathrm{d}\xi].$$

estimate of $\tau(t)$ as $\tau_1(t)$.

We are interested in exploring fractional order control laws such as

$$u(t) = -K_1 x(t) + K_2 \mathrm{D}_t^{\beta} x(t)$$

or even distributed order

$$u(t) = K \int_0^{\beta_{max}} c(\beta) \mathrm{D}_t^{\beta} x(t) \mathrm{d}\beta$$

where $c(\beta)$ is the weighting function and β_{max} is maximum order.

The dynamics of the estimated delay $\hat{\tau}(t)$ is generated from

$$D^{\beta}\hat{z}(t) = f_g(\hat{z}(t), u_{dg}(t)),\ \hat{z}(0) = \hat{z}_0,$$
$$\hat{\tau}(t) = h_g(\hat{z}(t), u_{dg}(t)).$$

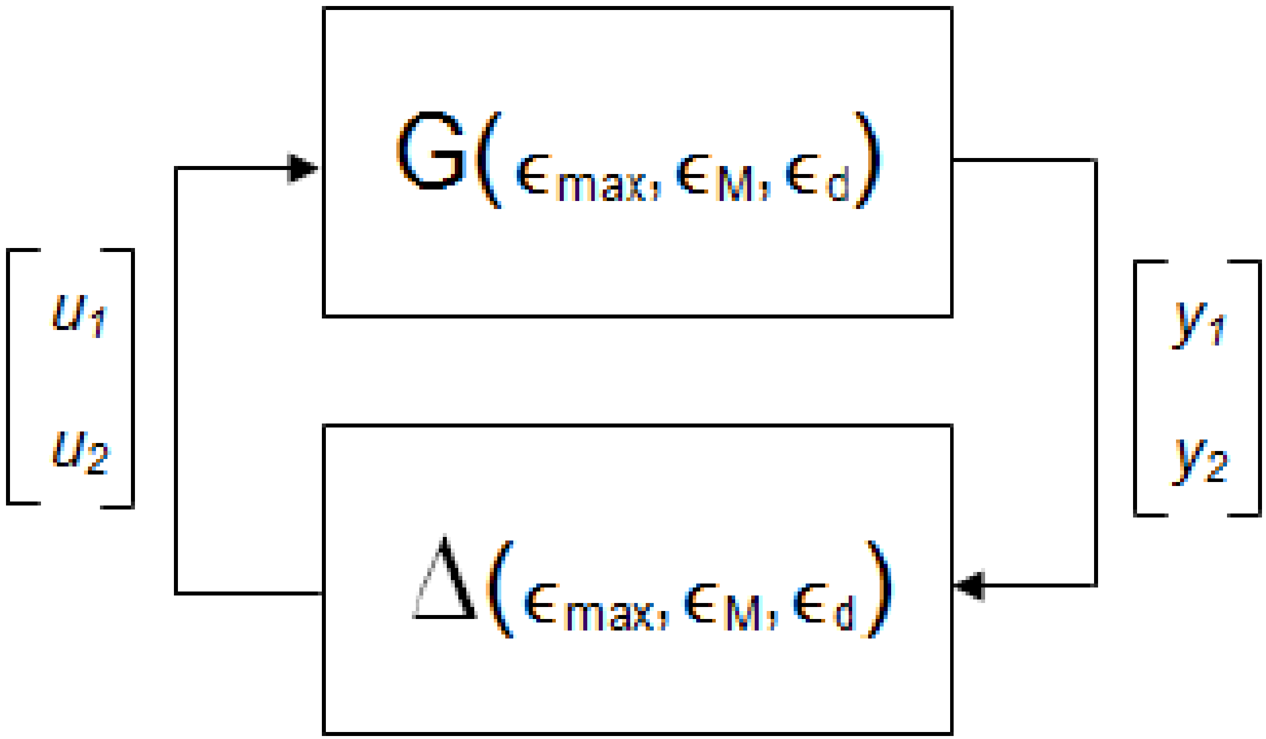
The signal $u_{dg}(t)$ and the functions $f_g(\cdot)$ and $h_g(\cdot)$ are assumed to be some known continuous functions in this normal case; $\hat{z}(t)$ is the internal state of the model. The estimated delay has the following property for all $t \geq 0$

$$0 \leq \hat{\tau}(t) \leq \hat{\tau}_{\max},$$

$$\{\epsilon_M, \epsilon_m\} \doteq \{\sup_t \epsilon(t), \inf_t \epsilon(t)\},\ \text{where } \epsilon(t) \doteq \tau(t) - \hat{\tau}(t).$$

$$
\begin{aligned}
u(t) = -K\{x(t) + \int_{t-\hat{\tau}_{\max}}^{t} & (t-\theta)^{\alpha-1} \\
\times E_{\alpha,\alpha}(A(t-\theta)^{\alpha})B[u(\theta) & - u(\theta-\hat{\tau}(\theta))]d\theta\} \\
= -Ky(t) + K\{\int_{t-\hat{\tau}_{\max}-\epsilon_{\max}}^{t-\hat{\tau}_{\max}} & (t-\theta)^{\alpha-1} \\
\times E_{\alpha,\alpha}(A(t-\theta)^{\alpha})B[u(\theta) & - u(\theta-\tau(\theta))]d\theta\} \\
+ \Delta_u(t), &
\end{aligned}
$$

$$
\begin{aligned}
\Delta_u(t) = K\{\int_{t-\hat{\tau}_{\max}}^{t} & (t-\theta)^{\alpha-1}E_{\alpha,\alpha}(A(t-\theta)^{\alpha}) \\
\times B[u(\theta-\hat{\tau}(\theta)) & - u(\theta-\tau(\theta))]d\theta\}.
\end{aligned}
$$

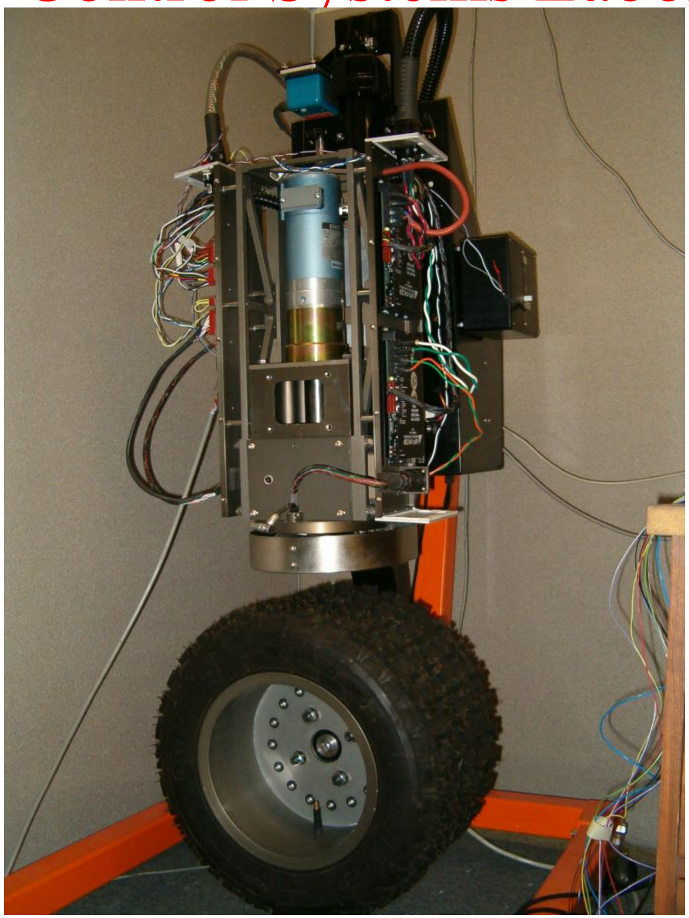


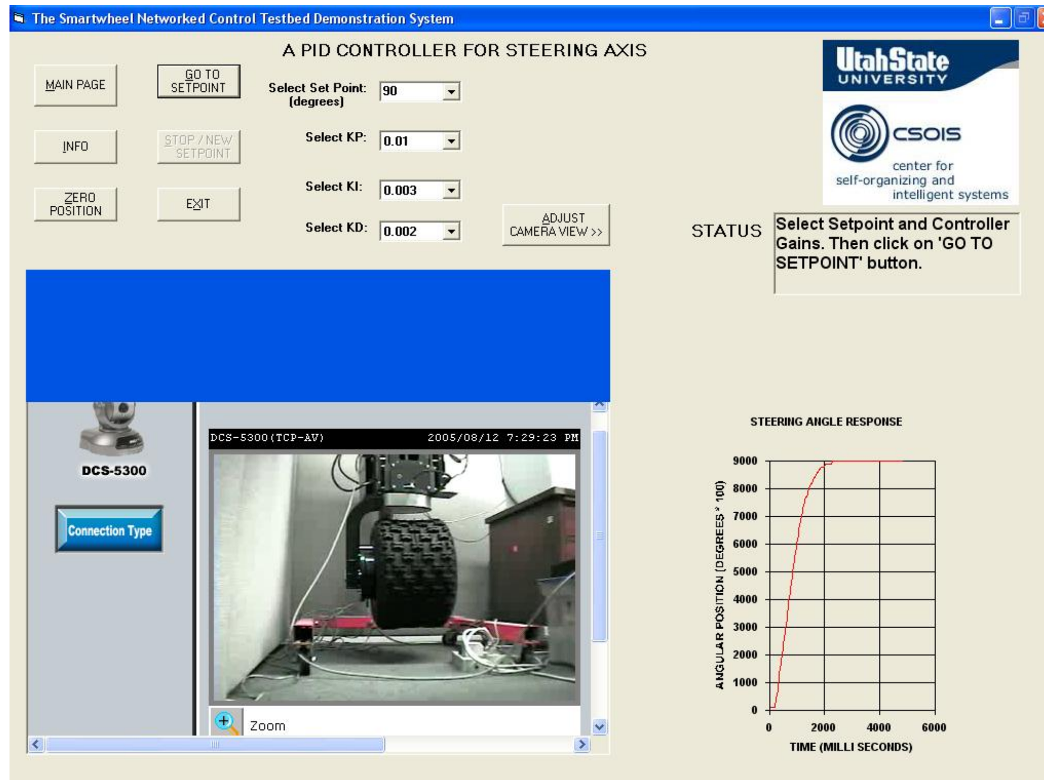
(b) Small gain formulation

Details: Xiaona Song, Ines Tejado and YangQuan Chen. "Remote Stabilization for Fractional-Order Systems via Communication Networks" ACC2010.

USU Smart Wheel Demo Rig

"Omni-directional Robotic Wheel - A Mobile Real-Time Control Systems Laboratory", Int. J. Eng. Edu. 2008.





http://www.csois.usu.edu/people/smartwheel/CompleteInfoPage.htm

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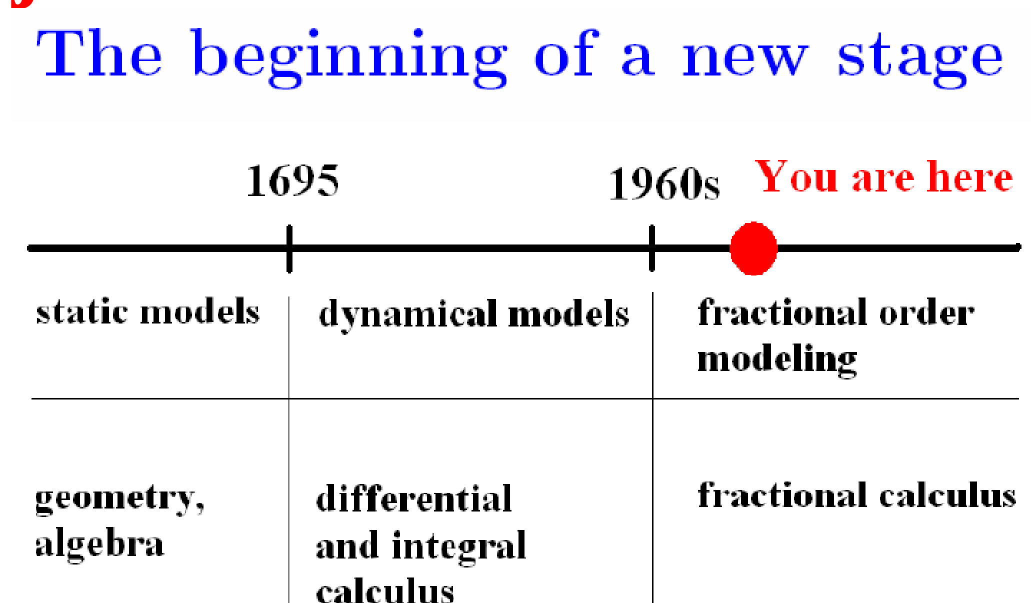
Concluding remarks

- "Go west, young man." – Horace Greeley
- "Go Fractional." – YangQuan Chen
- **Fractional Order Thinking enables exciting multidiscipline joint research that matters!**



Do more and do better!!

G.W. Scott Blair (1950):

"We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the system which we are studying."

S. Westerlund (1991):

"Expressed differently, we may say that Nature works with fractional time derivatives."

K. Nishimoto (1989):

"The fractional calculus is the calculus of the XXI century."

To probe further

http://www.tuke.sk/podlubny/
http://mechatronics.ece.usu.edu/foc/

Slide credit: Igor Podlubny

My submission - “Computational” can be put in front of almost every thing

- Computational intelligence
- Computational material
- Computational neuron science
- Computational psychology
- Computational fluid dynamic
- Computational biology
- Computational chemistry
- Computational ecology
- Computational social science
- Computational virology
- ….

My submission - “Control” can be put after almost every thing

- Speed Control
- Diet Control
- Weight Control
- Emotion Control
- Arm Control
- Microclimate Control
- Machine Control
- Human Gait Control
- Blood-pressure Control
- Aging Control
- Evacuation Control/Traffic Control/Conggestion Control
- ….

"Control Thinking"

- Computation is just a tool. Bringing "Systems Thinking" to computation is important due to the increased complexity
- Control has an objective in mind to change the system behavior to the desired one through feedback. To achieve the objective, the most important thing is the "purpose."
- Now, "signal-based" control is being replaced by "information-based" control. Need "Computational Thinking" yet with the "purpose" and "feedback" in mind.

So, here comes CPS

Computational Thinking +

Control Thinking + DPS =

==> Cyber Physical Systems

>= NCS

Fractional Order Thinking

- a.k.a "fractional order dynamic system thinking"
- Fractional order in either spatial evolution axis or temporal evolution axis.
- **Due to the complexity of the system, fractional thinking is essential to obtain insights and conclude rationally.**
- Bruce J. West. Where Medicine Went Wrong: Rediscovering the Path to Complexity. World Scientific Publishing Company. 2006. **ISBN-13:** 978-9812568830

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- Igor Podlubny, Ivo Petras, Lubomir Dorcak, Blas Vinagre, Shunji Manabe, J.T.M. Machado, J. Sabatier, Om Agrawal, Kevin L. Moore, Dingyu Xue, Anhong Zhou, Richard L. Magin, Wen Chen, Changpin Li, Yan Li.
- Concepción A. Monje, José Ignacio Suárez, Chunna Zhao, Jinsong Liang, Hyosung Ahn, Tripti Bhaskaran, Theodore Ndzana, Christophe Tricaud, Rongtao Sun, Nikita Zaveri, …